Contract Design for Aggregating, Trading, and Distributing Reserves in Demand-Side Frequency Regulation

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***Abstract*—With the integration of renewable energy sources to the power grid, the volatility of supply in the system will increase. Consequently, the mismatch between the power supply and demand may happen frequently and, thus, lead to frequency deviation from its nominal value. To avoid this scenario, demand-side flexibility has been widely considered to provide frequency regulation services. In this paper, we focus on the flexibility of thermal systems in buildings and propose a hierarchical demand-response market with a three-step algorithm to model the interactions among three entities: the independent system operators (ISOs), aggregators, and end users. The flexibility from the end users is aggregated in step 1, which is based on the incentive and electricity prices broadcasted by the aggregator. A robust optimization approach is adopted to improve the user's decision under the electricity price uncertainty. To model the interaction between the ISO and aggregators in step 2, a bilevel optimization problem is solved, in which the ISO seeks to minimize its cost, while the aggregators maximize their benefits in the day-ahead market. In step 3, each aggregator allocates its successful trading reserve among end users based on their performance scores.**

***Index Terms*—Aggregator, bilevel optimization, day-ahead market (DAM), demand response (DR), frequency regulation service (FRS), performance score.**

NOMENCLATURE

Indices

h	Index for time (Hours).
j	Index for aggregator, $j = 1, 2, \ldots, J$.
i	Index for buildings that are connected to an aggregator j, $i = 1, 2, \ldots, I_j$.
n	Index for reward, $n = 1, 2, \ldots, N$.

Parameters

$x_{i,j,h}$	Indoor temperature (°C).
$a_{i,j,h}$	Outdoor temperature (°C).
$m_{i,j}$	$+1$ for heating and -1 for cooling.
$\alpha_{i,j}, \beta_{i,j}$	Building insulation parameters.
$\tilde{p}_h$	Day-ahead electricity price (¢/kWh).
p_h^+, p_h^-	Positive/negative deviation of actual price from the forecasted price (¢/kWh).
$\delta_{i,j,h}$	End user's comfort satisfaction weight factor.
$r_{j,h}^+, r_{j,h}^-$	Up/down-reserve reward (¢/kWh).
$u_{i,j}^{\min}, u_{i,j}^{\max}$	Minimum/maximum bound of end users' power consumption (kW).
$x_{i,j,h}^+, x_{i,j,h}^-$	Up/down comfort temperature area (°C).
$\hat{x}_{i,j,h}$	Desired temperature level (°C).
$y_{i,j,h}^{\max}$	Maximum deviation of temperature (°C).
$\Gamma_{i,j}$	Conservativeness degree of decision maker regarding the price uncertainty.
$\overline{p}_h^g, \underline{p}_h^g$	Cost of procuring up/down reserve by generators (¢/kWh).
$\overline{W}_h, \underline{W}_h$	Up/down-reserve capacity the ISO aims to buy in the DAM (kWh).
$\eta_{i,j}$	End user's performance score.
$\hat{\eta}_{i,j}$	Adjusted performance score.
$\mathbf{M}$	Large positive constant.

Decision variables

$u_{i,j,h}$	Nominal power consumption of end users' thermal system (kWh).
$\upsilon_{i,j,h}^+, \upsilon_{i,j,h}^-$	Up/down-reserve capacity (kWh).
$\overline{E}_h^g, \underline{E}_h^g$	Generator up/down-reserve capacity (kWh).
$\overline{V}_{j,h}, \underline{V}_{j,h}$	Up/down-reserve capacity that the ISO buys from the aggregators (kWh).
$\overline{\lambda}_{j,h}, \underline{\lambda}_{j,h}$	Up/down-reserve price that the ISO pays to the aggregators (¢/kWh).
$y_{i,j,h}^+, y_{i,j,h}^-$	Up/down-reserve capacity that the aggregator buys from a building (kWh).

I. INTRODUCTION

THE frequency regulation service (FRS) facilitates the instantaneous balance between the demand and supply in the power grid and thus assists the independent system operators (ISOs) in maintaining the utility frequency within an acceptable

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range. Traditionally, the FRS is offered by the generators from the supply side. However, with the increase of the uncertainties due to the integration of renewable energy in the supply side and newly introduced mobile load such as electric vehicles in the demand side, maintaining the balance of the demand and supply on a second-by-second basis becomes much more challenging. As a result, the FRS not only requires a larger quantity of frequency regulation reserves, but also desires a higher quality of the reserves with the capacity of fast response. Based on recent studies [1]–[2], demand response (DR) has been acknowledged as a promising approach in offering flexibility to the power grid by controlling the power consumption of the flexible load and, hence, can offer the FRS as well. Moreover, such flexible loads can provide the FRS at a lower cost and without environmental impact compared to generators [3]–[4]. According to the Federal Energy Regulatory Commission (FERC) order 719, all the ISOs are required to accept bids from DR participants in their wholesale markets, and different ISOs can design and implement multiple DR strategies [5]. Among flexible loads, those with thermal storage, such as heating and cooling systems, in commercial and residential buildings have received significant consideration for the FRS due to their fast responsiveness [6]–[7]. The FERC issued a two-component market-based compensation scheme, FERC Order 755, which considers a capacity payment and a performance-based payment. The performance-based payment has incorporated by most of the U.S. ISOs, but it is measured and calculated in different ways for different ISOs (for example, California Independent System Operator (CAISO) measures the performance of frequency regulation by the accuracy percentage, which takes into account the deviations from the automatic generation control (AGC) signal [8]. In addition, the CAISO enforces the resources to be recertified for provision of the FRS if they violate a certain accuracy threshold (e.g., 50%), whereas PJM adopts a metric called performance score, which is computed based on the delay, correlation, and accuracy of the offered FRS [9]). In this work, we propose a three-step approach to design a contract for the CAISO day-ahead reserve market so that the demand-side FRS from the buildings can be obtained.

Because a single building's reserve for the FRS is too small (e.g., several kilowatts [10]) compared to the minimum reserve for participating in the FRS market (e.g., 0.5 MW in PJM [11]), an aggregator is typically needed to act as an agent between the ISO and the aggregate of buildings, which are willing to participate in the FRS [12]–[13]. The building automation system (BAS) is a computer-based control system installed in buildings and can act as a bridge between the aggregator and the end user for an intelligent management of energy. The BAS gathers data from sensors and relevant external information provider (e.g., aggregator) to maximize the operational performance of the facilities in buildings [14]–[15]. On the one hand, an aggregator has more expertise about the market process and bidding strategies and how to interact with all the BASs to aggregate their frequency regulation reserves. Several types of associations can be identified as aggregators, for instance local distributions companies, energy service companies, and electricity retailer. In this work, the aggregator is the retailer company that buys power from hundreds or thousands of buildings' owners and sells this power in the electricity market. It only provides financial incentives to the end users to participate in the FRS when possible and it does not have any control over the individual buildings. In particular, the aggregator determines the aggregate reserves of all the BASs and their corresponding prices, which are also known as quantity–price pairs. On the other hand, the aggregator trades the aggregate reserve in the FRS market, which is organized by the ISO, through submitting the quantity–price pairs as its bids. Based on the market clearing results, the accepted reserve will be awarded based on the corresponding price in the bid, and the aggregator distributes the reserve and rewards to the BASs. Note that for existing works that consider the participation of the aggregator in the FRS market, they assume that the aggregators are price takers and their bidding price is low enough so that all the bidding reserves will be accepted with the market clearing price. Thus, the aggregator only needs to estimate the maximal aggregate reserve value and neglects the price information for the bid [16]–[18], [23]. However, in practice, the price information is important for the BASs because each individually operated BAS will have different willingness to participate in such an FRS for different prices. Thus, it is still unclear that how the aggregator should obtain the aggregate price information and whether it is possible to strategically manipulate these bidding prices. In order to bridge this gap in the literature, we propose a contract between the aggregator and the BASs so that the aggregator can construct the quantity–price bids from the BASs explicitly, trades the reserve in the FRS market with the capability of manipulating the bidding prices, and distributes the cleared reserve and rewards to BASs proportionally according to their past performance so that a long-term high-quality participation from BASs is encouraged. In the proposed contract design, end users execute noncooperative strategies and keep their bidding information private. Moreover, we are limiting ourselves to the buildings that cannot easily participate in the FRS directly due to the minimum bid size requirement of the wholesale market.

II. Related Works and Contributions

There have been a number of research works on modeling residential DR and scheduling flexible appliances. The main aim of these studies is to minimize residential electricity payments [19]–[21] or to provide the FRS to the grid [22]. In [23], the authors considered an aggregate of commercial buildings and proposed a robust control framework for reliable scheduling of Heating, ventilation, and air conditioning (HVAC) systems against the uncertain frequency control signal sent by the ISO. Most of the proposed mechanisms are incentive based, where the customers receive rewards for their participation [24]–[26]. Hu *et al.* [26] introduced multilevel reward rates to encourage the residents to compromise outside their comfort zone area, especially if an emergency occurs for the grid, and the authors formulated an optimization problem to minimize total reward payment, while the customers' comfort level is maximized. In [27], the authors proposed an operational planning framework for large-scale thermal load dispatch. The aggregator estimates the maximum capacity of the loads, and the system operator performs a day-ahead scheduling, using a bilevel programming.

Incentive price information has an influence on users' participation in the FRS and their willingness to partially sacrifice their comfort for a reward. However, most of the initial works in this domain did not thoroughly discuss about the aggregator's bidding price and the BASs' responsiveness to different price information. In this work, we develop a comprehensive model with three hierarchical steps. Step 1 deals with the customers' day-ahead decisions in the energy market. In this stage, the aggregator broadcasts the electricity price and different profiles of reserve reward to the end users and asks for their reserve capacity to obtain the reserve from all the connected BASs in a discrete manner. After retrieving the information from an aggregation of the users, the ISO clears the market in step 2. A bilevel optimization method is applied with the objective of maximizing the aggregators' profits and minimizing the ISO's cost, which is a mixed-integer nonconvex problem. The idea of performance-based compensation for the FRS is considered in [28] and [29], in which the performance is appraised based on a real-time dispatch. In this work, we devise a new scheme, in which the aggregator contracts for the reserve capacity of end users based on their performance score and the ISO's need in the wholesale electricity market in step 3. Thus, if a user refuses to provide its promised flexibility in real time, it may be considered as an unreliable resource and the aggregator buys a lower level of flexibilities from it. Electricity prices announced in the day-ahead market (DAM) are subject to forecasting errors, which can influence the users' decision makings. Thus, it is prudent to design an algorithm that copes with such an uncertainty. The robust optimization approach has been paid great attention in practical applications in the last decades [30]–[32]. To this end, we apply the robust optimization method to the uncertain electricity price in this paper, and we specify the uncertain price by an uncertainty set, in which it can take values. In summary, this paper makes the following contributions:

1) a decision-making model for the aggregator is proposed to contract for demand-side reserves based on their reliability, following a three-step hierarchical framework;

2) providing the end users with different profiles of reserve rewards to construct the quantity–price bids from the BASs;

3) formulating a bilevel optimization problem to derive the optimal trading strategies between the ISO and aggregators in the electricity market and to simplify it to a single-level optimization problem;

4) considering the end users' performance scores when an aggregator needs to allocate its contracted reserve among different FRS participants;

5) considering the uncertainty associated with electricity prices and using a robust optimization technique to improve the decisions of BASs that act as FRS participants.

The organization of this paper is as follows. In Section III, a hierarchical FRS framework is explained as well as the mechanism design. In Section IV, a three-step algorithm with the corresponding mathematical formulation for each step is elaborated in detail. In Section V, we present numerical results based on simulations. Section VI concludes this paper.

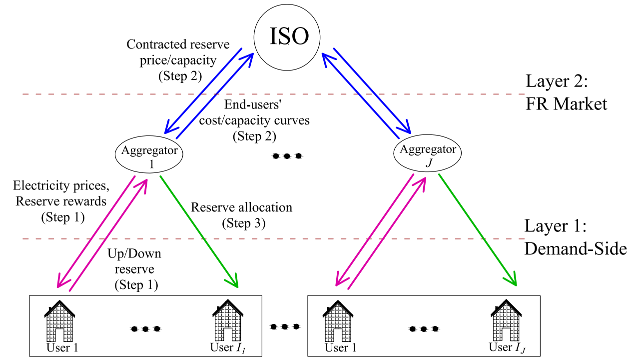

Fig. 1. Interaction of FRS participants—ISO, aggregators, and end users. Layer 1: Communication between the aggregators and building owners. Layer 2: Communication between the aggregators and the ISO.

III. Mechanism Design of the Hierarchical Market

The hierarchical structure of the demand-side FRS is illustrated in Fig. 1. At the lower level, the residents of a typical building can modify their demand profile according to an AGC signal advertised by the aggregator. The baseline demand pattern and the up/down-reserve capacity are calculated by a BAS that has taken into account the electricity price, monetary rewards, and users' comfort adjustment. In the upper level, the ISO has a contract with the aggregators and purchases the hourly capacity in the DAM. The wholesale energy market is comprised of a distinct DAM and real-time market (RTM). In the DAM, the ISO procures 100% of the predicted regulation requirements in hourly intervals. The real-lime market is a spot market, in which utilities can buy power to meet the last few increments of demand not covered in their day-ahead schedules. In this study, we model an aggregator that participates in the DAM and makes a contract with buildings' owners to provide sufficient FRS to the ISO the day after. We consider an electricity market consisting of an ISO, a set of aggregators $\mathcal{A} = \{1, 2, \ldots, J\}$, and a set of buildings $\mathcal{B}_j = \{1, 2, \ldots, I_j\}$ connected to an aggregator $j \in \mathcal{A}$, as the one depicted in Fig. 1. We consider a thermal system in each building, which has the flexibility to provide up/down reserve by increasing/shedding its energy consumption without significantly sacrificing end users' comfort. We assume that the residential buildings are equipped with the BAS technology, which facilitate the communication with the aggregator and the ISO for providing flexibility [33]. In-home display and smart thermostat technologies also facilitate a fine control of the buildings' thermal components. Let $i \in \mathcal{B}_j$ denote a building connected to an aggregator j; we adopt the temperature evolution dynamic equation $x_{i,j,h+1} = (1 - \alpha_{i,j})x_{i,j,h} + m_{i,j}\beta_{i,j}u_{i,j,h} + \alpha_{i,j}a_{i,j,h}$, where $h \in \mathcal{H}$ and $\mathcal{H}$ is the set of hours for the FRS. The mechanism of our contract design is as follows.

1) In the proposed model, we consider two layers, which are mathematically presented by a three-step algorithm. In layer 1, each building is connected to its registered aggregator, and in layer 2, the aggregators communicate with the ISO.

2) In step 1, the BAS makes a decision for an optimal reserve capacity in hourly intervals based on the information in

layer 1 such as electricity prices and reserve rewards. According to the contract, the utility company charges the end user for its reference power consumption, u_t, based on the real-time price irrespective of the deviation from the reference power consumption, which might be imposed by the regulation signal the next day. The end user gains rewards appropriate to the DAM reserve purchased by the aggregator.

3) In step 2, the contracts between the aggregators and the ISO are explored. The aggregators receive the up/down-reserve capacities of different connected buildings and participate in the bidding market with the aim of maximizing their revenues. The ISO purchases some reserve from each aggregator in the DAM for the FRS of the next operating day such that its cost is minimized. This step occurs in layer 2.

4) Step 3 addresses an allocation of reserve among users in layer 1. The aggregators take into account the historical performance of end users and accept more flexibilities from more reliable resources. We assume that the aggregator has direct access to measure users' responses to the AGC signal and to calculate each user's performance score.

IV. Problem Formulation and Algorithm

In this section, a three-step algorithm and an optimization problem related to each step are devised.

A. Step 1: Layer 1

The optimization problem of this stage is solved by the BAS on the user side. The decision variables are $\mathbf{DV_1}$:$\{u_{i,j,h}, \upsilon^+_{i,j,h}, \upsilon^-_{i,j,h}\}$. The optimal decision should be taken to not only reduce the total payment but also to capture the discomfort caused by the deviation from the reference pattern. The optimization problem for the BAS $i \in \mathcal{B}_j$ is

$$\mathbf{P_1}:$$

$$\min_{\mathbf{DV_1}} \sum_{h\in\mathcal{H}} \Big[\mathcal{C}_1\big(p_h, u_{i,j,h}\big) + \mathcal{C}_2\big(x_{i,j,h}, x^-_{i,j,h}, x^+_{i,j,h}, \hat{x}_{i,j,h}\big) - \mathcal{R}(\upsilon^-_{i,j,h}, \upsilon^+_{i,j,h}, r^-_{j,h}, r^+_{j,h})\Big]$$

subject to $(\forall h \in \mathcal{H})$:

$$x_{i,j,h+1} = A_{i,j}x_{i,j,h} + B_{i,j}u_{i,j,h} + w_{i,j,h} \tag{1a}$$

$$x^+_{i,j,h+1} = A_{i,j}x_{i,j,h} + B_{i,j}(u_{i,j,h} + \upsilon^+_{i,j,h}) + w_{i,j,h} \tag{1b}$$

$$x^-_{i,j,h+1} = A_{i,j}x_{i,j,h} + B_{i,j}(u_{i,j,h} - \upsilon^-_{i,j,h}) + w_{i,j,h} \tag{1c}$$

$$|x_{i,j,h} - \hat{x}_{i,j,h}| \le y^{\max}_{i,j,h} \tag{1d}$$

$$|x^+_{i,j,h} - \hat{x}_{i,j,h}| \le y^{\max}_{i,j,h} \tag{1e}$$

$$|x^-_{i,j,h} - \hat{x}_{i,j,h}| \le y^{\max}_{i,j,h} \tag{1f}$$

$$u^{\min}_{i,j} \le u_{i,j,h} \le u^{\max}_{i,j} \tag{1g}$$

$$u_{i,j,h} + \upsilon^+_{i,j,h} \le u^{\max}_{i,j}, \quad \upsilon^+_{i,j,h} \ge 0 \tag{1h}$$

$$u_{i,j,h} - \upsilon^-_{i,j,h} \ge u^{\min}_{i,j}, \quad \upsilon^-_{i,j,h} \ge 0 \tag{1i}$$

$$p_h \in \mathbf{P} \tag{1j}$$

where the cost of power consumption, the cost of discomfort, and the reserve payment are, respectively, $\mathcal{C}_1$, $\mathcal{C}_2$, and $\mathcal{R}$:

$$\mathcal{C}_1(p_h, u_{i,j,h}) = p_h u_{i,j,h}$$

$$\mathcal{C}_2(x_{i,j,h}, x^-_{i,j,h}, x^+_{i,j,h}, \hat{x}_{i,j,h}) = \delta_{i,j,h}[(x_{i,j,h} - \hat{x}_{i,j,h})^2 + (x^-_{i,j,h} - \hat{x}_{i,j,h})^2 + (x^+_{i,j,h} - \hat{x}_{i,j,h})^2]$$

$$\mathcal{R}(\upsilon^-_{i,j,h}, \upsilon^+_{i,j,h}, r^-_{j,h}, r^+_{j,h}) = r^-_{j,h}\upsilon^-_{i,j,h} + r^+_{j,h}\upsilon^+_{i,j,h}.$$

The cost of discomfort $\mathcal{C}_2$ is modeled as a summation of weighted quadratic distance between the indoor temperature $x_{i,j,h}$, up comfort temperature $x^+_{i,j,h}$, down comfort temperature $x^-_{i,j,h}$, and the end user's desired level of temperature $\hat{x}_{i,j,h}$ [34]. We call $\delta_{i,j,h} \ge 0$ the end user's comfort satisfaction weight factor that reflects the users discomfort caused by changing the demand from its desired amount. A smaller $\delta_{i,j,h}$ indicates that the end user is less sensitive to the comfort satisfaction and prefers to have a higher frequency regulation contribution. We assume that the knowledge on uncertainty is captured in an uncertainty set $\mathbf{P}$, which is bounded in the interval $[\tilde{p}_h - p^-_h, \tilde{p}_h + p^+_h]$. Deviations from the nominal power consumption are known as the up- and down-reserve capacity. Depending on the day-ahead contracted reserve, the ISO may change the control input within the lower envelope $(u_{i,j,h} - \upsilon^-_{i,j,h})$ and upper envelope $(u_{i,j,h} + \upsilon^+_{i,j,h})$. The range of $u_{i,j,h}$ is $[u^{\min}_{i,j}, u^{\max}_{i,j}]$ and time independent, in which $u^{\min}_{i,j}$ is usually close to zero and $u^{\max}_{i,j}$ is defined based on the technical specifications of the thermal system. Constraints (1a)–(1c) are the state-space modeling of the building thermal system, where $A_{i,j} = (1 - \alpha_{i,j})$, $B_{i,j} = m_{i,j}\beta_{i,j}$, and $w_{i,j,h} = \alpha_{i,j}a_{i,j,h}$, and they demonstrate how the current state $x_{i,j,h}$ (temperature in this paper), control input $\{u_{i,j,h}, u_{i,j,h} + \upsilon^+_{i,j,h}, u_{i,j,h} - \upsilon^-_{i,j,h}\}$, and weather condition $w_{i,j,h}$ affect the state in the future time step. The aggregator provides a number of up/down-reserve rewards pairs $\{r^+_{j,h}, r^-_{j,h}\}$ to each connected end user in day-ahead. Then, each end user i solves the optimization problem P_1 for each reward pair $\{r^+_{j,h}, r^-_{j,h}\}$ and obtains the optimal reserve capacity $\{\upsilon^+_{i,j,h}, \upsilon^-_{i,j,h}\}$. After collecting the outcomes of all the optimization problems solved by all the end users, the aggregator can generate a database of $\{\upsilon^+_{i,j,h}, \upsilon^-_{i,j,h}, r^+_{j,h}, r^-_{j,h}\}$, in which the optimal reserve capacity of all the end users for each reward pair is listed. In Step 3, the database is used to find the best contract between the aggregator and BASs.

Robust optimization is an approach to deal with the uncertainty in optimization problems especially when the probability distribution function of the uncertainty cannot be easily described. Since the actual concern of the user is the electricity price being more than the forecasted value, we study the case that the actual price is more than the prediction. Uncertainty modeling of the price can be expressed as

$$p_h = \tilde{p}_h + p^+_h \epsilon_{i,j,h}, \qquad 0 \le \epsilon_{i,j,h} \le 1. \tag{2}$$

For a compact representation of the objective, we define $\boldsymbol{x_{i,j,h}} = (x_{i,j,h}, x^-_{i,j,h}, x^+_{i,j,h}, \hat{x}_{i,j,h})$, $\boldsymbol{\upsilon_{i,j,h}} = (\upsilon^-_{i,j,h}, \upsilon^+_{i,j,h})$, $\boldsymbol{r_{j,h}} = (r^-_{j,h}, r^+_{j,h})$, and $\mathcal{J}(\boldsymbol{x_{i,j,h}}, \boldsymbol{\upsilon_{i,j,h}}, \boldsymbol{r_{j,h}}) = \mathcal{C}_2(\boldsymbol{x_{i,j,h}}) - \mathcal{R}(\boldsymbol{\upsilon_{i,j,h}}, \boldsymbol{r_{j,h}})$. The last term will be used in the objective function for the rest of the paper. By inserting (2) into problem $\mathbf{P_1}$, the new problem is expressed as

$$\acute{\mathbf{P}}_\mathbf{1}:$$

$$\begin{aligned}
\min_{\mathbf{DV_1}} \quad & \sum_{h\in\mathcal{H}} \big[\tilde{p}_h u_{i,j,h} + p^+_h \epsilon_{i,j,h} u_{i,j,h} \\
& + \mathcal{J}\big(\boldsymbol{x_{i,j,h}}, \boldsymbol{\upsilon_{i,j,h}}, \boldsymbol{r_{j,h}}\big)\big] \\
\textbf{subject to} \quad & \sum_{h\in\mathcal{H}} \epsilon_{i,j,h} \le \Gamma_{i,j} \quad , \quad 0 \le \epsilon_{i,j,h} \le 1 \quad \forall h \in \mathcal{H} \qquad \text{(3a)} \\
& \text{(1a)–(1i)} \quad \forall h \in \mathcal{H}.
\end{aligned}$$

$\Gamma_{i,j}$ is the uncertain coefficient that can be tolerated and it allows the decision maker to control the degree of conservatism of the solution. Taking a value of $\Gamma_{i,j}$ between 0 and 24 allows the decision maker to achieve a tradeoff between the nominal performance of the deterministic model and the risk protection of the most conservative model. According to [35], using the robust counterpart of problem $\acute{\mathbf{P}}_\mathbf{1}$ and Lagrangian duality, problem $\acute{\mathbf{P}}_\mathbf{1}$ can be written into

$$\acute{\acute{\mathbf{P}}}_\mathbf{1}:$$

$$\begin{aligned}
\min_{\mathbf{DV_1}, \xi_{i,j,h}, \mu_{i,j}} \quad & \sum_{h\in\mathcal{H}} \big[\tilde{p}_h u_{i,j,h} + \mathcal{J}(\boldsymbol{x_{i,j,h}}, \boldsymbol{\upsilon_{i,j,h}}, \boldsymbol{r_{j,h}})\big] \\
& + \mu_{i,j}\Gamma_{i,j} \\
\textbf{subject to} \quad & (\forall h \in \mathcal{H}): \\
& \mu_{i,j} + \xi_{i,j,h} \ge p^+_h u_{i,j,h}, \quad \mu_{i,j}, \xi_{i,j,h} \ge 0 \\
& \text{(1a)–(1i)} \qquad \text{(4a)}
\end{aligned}$$

where $\xi_{i,j,h}$ and $\mu_{i,j}$ are the Lagrange multipliers of the inequality constraint (3a). The simplified optimization problem $\acute{\acute{\mathbf{P}}}_\mathbf{1}$ is convex, and it can be solved efficiently using the optimization software package CVX.

B. Step 2: Layer 2

In layer 2, the participant aggregators offer their reserve capacity to the ISO, which is obtained from each reward profile $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ in step 1. In this paper, we select a bilevel program to model the decision process between the aggregators and the ISO in a hierarchical structure. One benefit of using such bilevel programming is its transformation to a single-level optimization problem that can be solved by the commercially available software. A distinguishing characteristic of multilevel systems is that the decision maker at one level may be able to influence the behavior of a decision maker at another level but not completely control his actions [36]. In the proposed model, the ISO aims at minimizing its overall cost as shown in P_3, while the aggregators plan to maximize their own revenues in the trading market. The problems P_2 and P_3 model the following decisions process: The aggregators first announce their up/down-reserve prices $[\overline{\lambda}_{j,h}, \underline{\lambda}_{j,h}]$, which reflect their own valuations on their offered reserve capacity, to the ISO with the aim to maximize their own revenues. The ISO collects all the claimed reserve prices from all the aggregators, based on which the ISO determines the optimal reserve capacity of each aggregator. Given $\overline{\mathbf{V}}_{j,h,n} = \sum_{i\in\mathcal{B}_j} \upsilon^+_{i,j,h}$ and $\underline{\mathbf{V}}_{j,h,n} = \sum_{i\in\mathcal{B}_j} \upsilon^-_{i,j,h}$, note that $\upsilon^+_{i,j,h}$ and $\upsilon^-_{i,j,h}$ are obtained from step 1, and they depend on $r^+_{j,h,n}$ and $r^-_{j,h,n}$. Let $a_{j,n} \in \{0, 1\}$ be the decision variable of aggregator j, indicating which reserve reward profile n is chosen by the aggregator and thus how much up- and down-reserve capacities can be offered to the ISO. Then, the reserve market clearing prices and capacities are obtained by solving the following bilevel optimization problem:

$$\mathbf{P_2}:$$

$$\begin{aligned}
\max_{\mathbf{DV_2}} \quad & \sum_{h\in\mathcal{H}} \sum_{j\in\mathcal{A}} \{\overline{\lambda}_{j,h} \overline{V}_{j,h} + \underline{\lambda}_{j,h} \underline{V}_{j,h} - \overline{f}_c(\overline{V}_{j,h}) \\
& - \underline{f}_c(\overline{V}_{j,h})\} \\
\textbf{subject to} \quad & \sum_{n\in\mathcal{N}} a_{j,n} = 1 \quad \forall j \in \mathcal{A} \qquad \text{(5a)} \\
& \overline{V}_{j,h}, \underline{V}_{j,h} \in \quad \arg\{\mathbf{P_3}\} \quad \forall h \in \mathcal{H}, \forall j \in \mathcal{A} \qquad \text{(5b)}
\end{aligned}$$

$$\mathbf{P_3}:$$

$$\begin{aligned}
\min_{\mathbf{DV_3}} \quad & \sum_{h\in\mathcal{H}} \left\{ \overline{p}^g_h \overline{E}^g_h + \underline{p}^g_h \underline{E}^g_h + \sum_{j\in\mathcal{A}} (\overline{\lambda}_{j,h} \overline{V}_{j,h} + \underline{\lambda}_{j,h} \underline{V}_{j,h}) \right\} \\
\textbf{subject to} \quad & (\forall h \in \mathcal{H}, \forall j \in \mathcal{A}, \forall n \in \mathcal{N}): \\
& \overline{E}^g_h + \sum_{j\in\mathcal{A}} \overline{V}_{j,h} \ge \overline{W}_h \qquad \text{(6a)} \\
& \underline{E}^g_h + \sum_{j\in\mathcal{A}} \underline{V}_{j,h} \ge \underline{W}_h \qquad \text{(6b)} \\
& 0 \le \overline{E}^g_h \le \overline{E}^{\max}_h \qquad \text{(6c)} \\
& 0 \le \underline{E}^g_h \le \underline{E}^{\max}_h \qquad \text{(6d)} \\
& 0 \le \acute{\overline{V}}_{j,h,n} \le a_{j,n} \overline{\mathbf{V}}_{j,h,n} \qquad \text{(6e)} \\
& 0 \le \acute{\underline{V}}_{j,h,n} \le a_{j,n} \underline{\mathbf{V}}_{j,h,n} \qquad \text{(6f)} \\
& \sum_{n\in\mathcal{N}} \acute{\overline{V}}_{j,h,n} = \overline{V}_{j,h} \qquad \text{(6g)} \\
& \sum_{n\in\mathcal{N}} \acute{\underline{V}}_{j,h,n} = \underline{V}_{j,h} \qquad \text{(6h)}
\end{aligned}$$

where the decision variables are $\mathbf{DV_2}$:$\{\overline{\lambda}_{j,h}, \underline{\lambda}_{j,h}, a_{j,n}\}$ and $\mathbf{DV_3}$:$\{\overline{E}^g_h, \underline{E}^g_h, \acute{\overline{V}}_{j,h,n}, \acute{\underline{V}}_{j,h,n}, \overline{V}_{j,h}, \underline{V}_{j,h}\}$. The upper level problem $\mathbf{P_2}$ represents the profit maximization of the aggregators. The individual aggregator's estimation of costs for purchasing up and down reserves from the users are, respectively, denoted by $\overline{f}_c(\overline{V}_{j,h})$ and $\underline{f}_c(\underline{V}_{j,h})$ as follows:

$$\overline{f}_c(\overline{V}_{j,h}) = \sum_{n\in\mathcal{N}} r^{+}_{j,h,n}\acute{\overline{V}}_{j,h,n}\,, \quad \underline{f}_c(\underline{V}_{j,h}) = \sum_{n\in\mathcal{N}} r^{-}_{j,h,n}\acute{\underline{V}}_{j,h,n}.$$

The lower level problem represents the market clearing with the aim of minimizing the cost of the ISO. The hourly up/down reserve the ISO buys in the DAM can be provided by either the aggregators or the generator. For a simple representation of mathematical problem $\mathbf{P_3}$, we use two auxiliary decision variables $\acute{\overline{V}}_{j,h,n}$ and $\acute{\underline{V}}_{j,h,n}$, and their relationships with $\overline{V}_{j,h}$ and $\underline{V}_{j,h}$ are defined in constraints (6g) and (6h), respectively. We assume that the ISO decides to purchase a fixed amount of reserve in the DAM, and we neglect what it may buy in the RTM to compensate for the uncertain level of imbalances between the supply and demand. Constraints (6a) and (6b) ensure that the amount of reserves required by the ISO is procured. Thus, the solution of problems $\mathbf{P_2}$ and $\mathbf{P_3}$ clarifies the up- and down-reserve contract between the ISO and each aggregator j based on the reserve price and capacity. Since the lower level problem is linear, one approach to simplify the bilevel optimization problems is to replace the lower level problem by its Karush–Kuhn–Tucker (KKT) conditions. Thus

$\acute{\mathbf{P}}_\mathbf{2}$:

$$\max_{\mathbf{DV_4}} \sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{A}}\{\overline{\lambda}_{j,h}\overline{V}_{j,h} + \underline{\lambda}_{j,h}\underline{V}_{j,h} - \overline{f}_c(\overline{V}_{j,h}) - \underline{f}_c(\underline{V}_{j,h})\}$$

subject to $(\forall h \in \mathcal{H}, \forall j \in \mathcal{A}, \forall n \in \mathcal{N})$:

$$\underline{\lambda}_{j,h} - \underline{\epsilon}_h + \underline{\varrho}_{j,h} \geq 0 \tag{7a}$$

$$\overline{\lambda}_{j,h} - \overline{\epsilon}_h + \overline{\varrho}_{j,h} \geq 0 \tag{7b}$$

$$\underline{p}^g_h - \underline{\epsilon}_h + \underline{\varphi}_h \geq 0 \tag{7c}$$

$$\overline{p}^g_h - \overline{\epsilon}_h + \overline{\varphi}_h \geq 0 \tag{7d}$$

$$\underline{\epsilon}_h\left(\underline{W}_h - \underline{E}^g_h - \sum_{j\in\mathcal{A}}\underline{V}_{j,h}\right) = 0 \tag{7e}$$

$$\overline{\epsilon}_h\left(\overline{W}_h - \overline{E}^g_h - \sum_{j\in\mathcal{A}}\overline{V}_{j,h}\right) = 0 \tag{7f}$$

$$\underline{\varrho}_{j,h}\left(\acute{\underline{V}}_{j,h,n} - a_{j,n}\underline{\mathbf{V}}_{j,h,n}\right) = 0 \tag{7g}$$

$$\overline{\varrho}_{j,h}\left(\acute{\overline{V}}_{j,h,n} - a_{j,n}\overline{\mathbf{V}}_{j,h,n}\right) = 0 \tag{7h}$$

$$\underline{\varphi}_h(\underline{E}^g_h - \underline{E}^{\max}_h) = 0 \tag{7i}$$

$$\overline{\varphi}_h(\overline{E}^g_h - \overline{E}^{\max}_h) = 0 \tag{7j}$$

$$\underline{\pi}_{j,h}\acute{\underline{V}}_{j,h,n} = 0 \tag{7k}$$

$$\overline{\pi}_{j,h}\acute{\overline{V}}_{j,h,n} = 0 \tag{7l}$$

$$\underline{\kappa}_h\underline{E}^g_h = 0 \tag{7m}$$

$$\overline{\kappa}_h\overline{E}^g_h = 0 \tag{7n}$$

$$\underline{\epsilon}_h, \overline{\epsilon}_h, \underline{\pi}_{j,h}, \overline{\pi}_{j,h}, \underline{\kappa}_h, \overline{\kappa}_h, \underline{\varphi}_h, \overline{\varphi}_h, \underline{\varrho}_{j,h}, \overline{\varrho}_{j,h} \geq 0 \tag{7o}$$

$$\text{(5a), (6a)–(6h).} \tag{7p}$$

The decision variables are defined by $\mathbf{DV_4}$:$\{\mathbf{DV_2}, \mathbf{DV_3}, \underline{\epsilon}_h, \overline{\epsilon}_h, \underline{\pi}_{j,h}, \overline{\pi}_{j,h}, \underline{\kappa}_h, \overline{\kappa}_h, \underline{\varphi}_h, \overline{\varphi}_h, \underline{\varrho}_{j,h}, \overline{\varrho}_{j,h}\}$. We refer to the variables in constraint (7o) as the KKT multipliers of the follower problem $\mathbf{P_3}$. The transformed problem $\acute{\mathbf{P}}_\mathbf{2}$ is a mixed-integer nonconvex problem due to the products of variables in the objective and constraints. In the following part, we will show how to transform this problem into a mixed-integer linear program (MILP), which can be solved efficiently by commercial solvers. The first difficulty originates from the complementary slackness conditions as they consist of products of variables. We follow the procedure in [41] to transform problem $\acute{\mathbf{P}}_\mathbf{2}$ into an equivalent mixed-integer problem. The new decision variables are defined by $\mathbf{DV_5}$:$\{\mathbf{DV_4}, \overline{\alpha}_{j,h}, \underline{\alpha}_{j,h}, \overline{\beta}_{j,h}, \underline{\beta}_{j,h}, \bar{\mathrm{a}}_h, \underline{\mathrm{a}}_h, \bar{\mathrm{b}}_h, \underline{\mathrm{b}}_h, \overline{\chi}_{j,h}, \underline{\chi}_{j,h}\}$.

$\acute{\acute{\mathbf{P}}}_\mathbf{2}$:

$$\max_{\mathbf{DV_5}} \sum_{h\in\mathcal{H}}\{\overline{\epsilon}_h\overline{W}_h + \underline{\epsilon}_h\underline{W}_h - \overline{p}^g_h\overline{E}^g_h - \underline{p}^g_h\underline{E}^g_h - \overline{\varphi}_h\overline{E}^{\max}_h - \underline{\varphi}_h\underline{E}^{\max}_h\} - \sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{A}}\{\overline{f}_c(\overline{V}_{j,h}) + \underline{f}_c(\underline{V}_{j,h})\} - \sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{A}}\sum_{n\in\mathcal{N}} a_{j,n}(\overline{\varrho}_{j,h}\overline{\mathbf{V}}_{j,h,n} + \underline{\varrho}_{j,h}\underline{\mathbf{V}}_{j,h,n})$$

subject to $(\forall h \in \mathcal{H}, \forall j \in \mathcal{A})$:

$$(a_{j,n}\overline{\mathbf{V}}_{j,h,n} - \acute{\overline{V}}_{j,h,n}) \leq (1 - \overline{\chi}_{j,h})\mathbf{M} \tag{8a}$$

$$\overline{\varrho}_{j,h} \leq \mathbf{M}\overline{\chi}_{j,h} \tag{8b}$$

$$(a_{j,n}\underline{\mathbf{V}}_{j,h,n} - \acute{\underline{V}}_{j,h,n}) \leq (1 - \underline{\chi}_{j,h})\mathbf{M} \tag{8c}$$

$$\underline{\varrho}_{j,h} \leq \mathbf{M}\underline{\chi}_{j,h} \tag{8d}$$

$$(\overline{\lambda}_{j,h} - \overline{\epsilon}_h + \overline{\varrho}_{j,h}) \leq \overline{\alpha}_{j,h}\mathbf{M} \tag{8e}$$

$$\overline{V}_{j,h} \leq (1 - \overline{\alpha}_{j,h})\mathbf{M} \tag{8f}$$

$$(\underline{\lambda}_{j,h} - \underline{\epsilon}_h + \underline{\varrho}_{j,h}) \leq \underline{\alpha}_{j,h}\mathbf{M} \tag{8g}$$

$$\underline{V}_{j,h} \leq (1 - \underline{\alpha}_{j,h})\mathbf{M} \tag{8h}$$

$$(\overline{p}^g_h - \overline{\epsilon}_h + \overline{\varphi}_h) \leq \overline{\beta}_{j,h}\mathbf{M} \tag{8i}$$

$$\overline{E}^g_h \leq (1 - \overline{\beta}_{j,h})\mathbf{M} \tag{8j}$$

$$(\underline{p}^g_h - \underline{\epsilon}_h + \underline{\varphi}_h) \leq \underline{\beta}_{j,h}\mathbf{M} \tag{8k}$$

$$\underline{E}^g_h \leq (1 - \underline{\beta}_{j,h})\mathbf{M} \tag{8l}$$

$$\overline{\varphi}_h \leq \bar{\mathrm{a}}_h\mathbf{M} \tag{8m}$$

$$(\overline{E}^{\max}_h - \overline{E}^g_h) \leq (1 - \bar{\mathrm{a}}_h)\mathbf{M}[-2pt] \tag{8n}$$

$$\underline{\varphi}_h \le \underline{\mathrm{a}}_h \mathbf{M} \tag{8o}$$
$$(\underline{E}_h^{\max} - \underline{E}_h^{g}) \le (1-\underline{\mathrm{a}}_h)\mathbf{M} \tag{8p}$$
$$\overline{\epsilon}_h \le \overline{\mathrm{b}}_h \mathbf{M} \tag{8q}$$
$$\left(\overline{E}_h^{g} + \sum_{j\in\mathcal{A}} \overline{V}_{j,h} - \overline{W}_h\right) \le (1-\overline{\mathrm{b}}_h)\mathbf{M} \tag{8r}$$
$$\underline{\epsilon}_h \le \underline{\mathrm{b}}_h \mathbf{M} \tag{8s}$$
$$\left(\underline{E}_h^{g} + \sum_{j\in\mathcal{A}} \underline{V}_{j,h} - \underline{W}_h\right) \le (1-\underline{\mathrm{b}}_h)\mathbf{M} \tag{8t}$$
$$\overline{\alpha}_{j,h}, \underline{\alpha}_{j,h}, \overline{\beta}_{j,h}, \underline{\beta}_{j,h}, \overline{\mathrm{a}}_h, \underline{\mathrm{a}}_h, \overline{\mathrm{b}}_h, \underline{\mathrm{b}}_h, \overline{\chi}_{j,h}, \underline{\chi}_{j,h}, \in \{0,1\}$$
$$(5a), (6a)\text{–}(6h), (7a)\text{–}(7d), (7o). \tag{8u}$$

Next, we continue to linearize the nonlinear objective and equivalently transform problem $\acute{\mathbf{P}}_2$ into a standard MILP problem. Let $\overline{z}_{j,h,n} = a_{j,n}\overline{\varrho}_{j,h}$ and $\overline{\varrho}^L_{j,h} \le \overline{\varrho}_{j,h} \le \overline{\varrho}^U_{j,h}$; we can define the following constraints:

$$\overline{z}_{j,h,n} \le \overline{\varrho}^U_{j,h} a_{j,n} \tag{9}$$
$$\overline{z}_{j,h,n} \ge \overline{\varrho}^L_{j,h} a_{j,n} \tag{10}$$
$$\overline{z}_{j,h,n} \le \overline{\varrho}_{j,h} - \overline{\varrho}^L_{j,h}(1-a_{j,n}) \tag{11}$$
$$\overline{z}_{j,h,n} \ge \overline{\varrho}_{j,h} - \overline{\varrho}^U_{j,h}(1-a_{j,n}) \tag{12}$$

where $\overline{\varrho}^U_{j,h}$ and $\overline{\varrho}^L_{j,h}$ are proper upper and lower bounds for $\overline{\varrho}_{j,h}$. The equivalence between (9)–(12) and $\overline{z}_{j,h,n} = a_{j,n}\overline{\varrho}_{j,h}$ can be checked as follows: If $a_{j,n} = 0$, (9) and (10) force $\overline{z}_{j,h,n} = 0$, which is true. Equations (11) and (12) can also satisfy $\overline{z}_{j,h,n} = 0$. If $a_{j,n} = 1$, then $\overline{z}_{j,h,n} = \overline{\varrho}_{j,h}$, which is satisfied in (9)–(12). The linearization for $a_{j,n}\underline{\varrho}_{j,h}$ can be written in the same way, which we skip for the sake of brevity. The final optimization problem is the MILP, which can be solved efficiently by commercial solvers (e.g., Gurobi). MILP problems are generally solved using a linear-programming-based branch-and-bound algorithm [37].

C. Step 3: Layer 1

After retrieving information from the BAS in the aggregation of buildings and finalizing the contract between the ISO and aggregators, the following optimization problem is solved on the aggregator side to allocate the reserve capacity among the end users. In step 3, the objective function is the cost of the up and down reserves that an aggregator purchases from its registered users over the total time horizon. The decision variable $\mathbf{DV_6}$:$\{y^+_{i,j,h}, y^-_{i,j,h}\}$ includes the amount of up and down reserves bought from each participating building i. Given the value of $a_{j,n}$ in step 2, let $a_{j,n^*} = 1$ and $a_{j,n\neq n^*} = 0$. Then, each aggregator searches in its database to obtain the appropriate reserve rewards r^+_{j,h,n^*} and r^-_{j,h,n^*}, and their corresponding capacities $\upsilon^+_{i,j,h}$ and $\upsilon^-_{i,j,h}$ for constraints (13c) and (13d). Constraints (13a) and (13b) enforce the aggregate flexibilities by all the buildings to be equal to a certain amount denoted by $\overline{V}_{j,h}$ and $\underline{V}_{j,h}$ and the ISO aims to achieve them according to the contract with an aggregator j in step 2. In the objective, $q_{i,j}$ is a weight factor that shows the FRS cost score of building i connected to an aggregator j

$$\mathbf{P_4}:$$
$$\min_{\mathbf{DV_6}} \quad \sum_{h\in\mathcal{H}} \sum_{i\in\mathcal{B}_j} q_{i,j}\left(r^+_{j,h,n^*} y^+_{i,j,h} + r^-_{j,h,n^*} y^-_{i,j,h}\right)$$
$$\textbf{subject to} \quad (\forall h \in \mathcal{H}, \forall i \in \mathcal{B}_j):$$
$$\sum_{i\in\mathcal{B}_j} y^+_{i,j,h} = \overline{V}_{j,h} \tag{13a}$$
$$\sum_{i\in\mathcal{B}_j} y^-_{i,j,h} = \underline{V}_{j,h} \tag{13b}$$
$$0 \le y^+_{i,j,h} \le \upsilon^+_{i,j,h} \tag{13c}$$
$$0 \le y^-_{i,j,h} \le \upsilon^-_{i,j,h}. \tag{13d}$$

The weight factor $q_{i,j}$ defined in the objective function of problem $\mathbf{P_4}$ is adopted based on the adjusted performance score, say $\hat{\eta}_{i,j}$, which is simply characterized by the absolute deviation of the resource's response (e.g., $\acute{y}_{i,j,t}$) from the real-time up/down regulation signal (e.g., $\tilde{y}_{i,j,t}$) it needs to provide at time t. Let $t \in \mathcal{T} = \{1, 2, \ldots, 24 \times 60\}$ denote the minute-by-minute time scale in which the absolute deviation $D_{i,j,t}$ is measured by $D_{i,j,t} = |\tilde{y}_{i,j,t} - \acute{y}_{i,j,t}|$. Thus, the daily performance score of each participant is calculated by

$$\eta_{i,j} = \mathbf{max}\left\{0, 1 - \frac{\sum_{t\in\mathcal{T}} D_{i,j,t}}{\sum_{t\in\mathcal{T}} \tilde{y}_{i,j,t}}\right\}. \tag{14}$$

If a resource follows the AGC signal accurately, its performance score equals 1; otherwise, it equals a value less than 1. It is important to smooth the performance score over time so that the resource is not instantaneously penalized. The adjusted performance score considers the history of the resource in previous days and it may be characterized by $\hat{\eta}_{i,j} = (1-k)\overline{\eta}_{i,j} + k\eta_{i,j}$, where $\overline{\eta}_{i,j}$ is the history of the performance score of resource i and k is a value between 0 and 1. Based on this definition, a resource with a low adjusted performance score $\hat{\eta}_{i,j}$ may impose some cost to the aggregator. Therefore, in layer 1, the weight factor of each user in the objective equals $q_{i,j} = 1 - \hat{\eta}_{i,j}$, which we call the cost score since the objective is a minimization problem.

V. Numerical Results

In this section, we simulate the model for four heterogeneous aggregators, each of which is connected to a set of 1000 heterogeneous buildings with different reserve capacities, desired temperature, comfort satisfaction weight factor, and performance scores. We assume each building has a thermal system with a reference temperature between 19 to 22 °C and a maximum allowed deviation of 2 °C [38]–[40]. The model has been solved using CVX with a Gurobi solver to deal with the binary decision variables.

A. Performance Analysis in Step 1

The BAS considers the electricity price uncertainty for making decisions in step 1. Therefore, sensitivity analysis is provided in this section in terms of volatility in the DAM prices for a

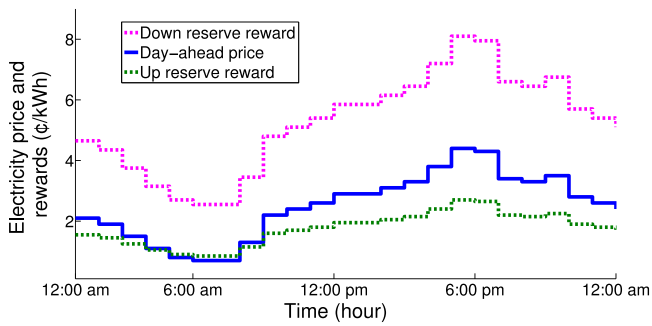

Fig. 2. Day-ahead electricity price and hourly rewards for up/down reserve.

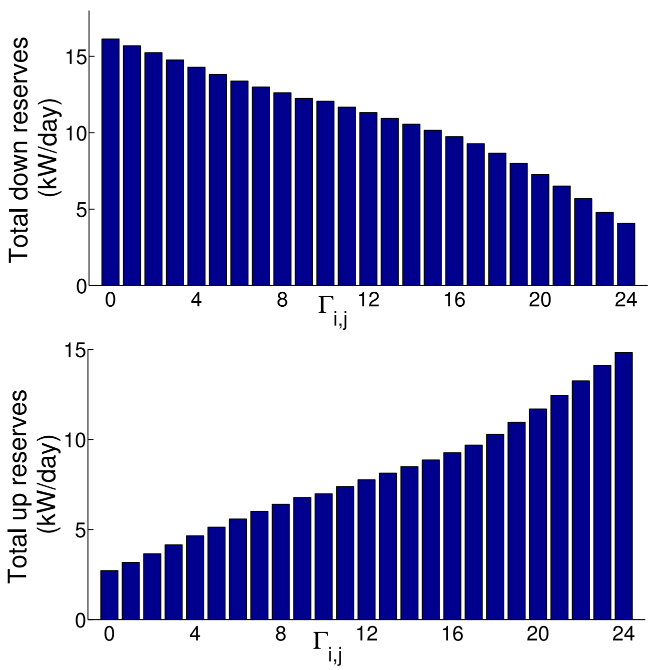

Fig. 3. Total up/down daily reserve for different $\Gamma_{i,j}$.

typical building i that is connected to aggregator j. We solve the deterministic and robust optimization problems and compare the results. Hourly up/down-reserve rewards and the day-ahead price used for the simulations are shown in Fig. 2. The simulation was carried out for different values of $\Gamma_{i,j}$ to explore the residents' behavior against different levels of price uncertainty. The results in Fig. 3 indicate that, with higher values of the $\Gamma_{i,j}$ coefficient, users tend to offer less down reserve but more up reserve. Fig. 4 demonstrates the user's decisions regarding the reference power consumption and reserve capacity when there is no uncertainty in the electricity price ($\Gamma_{i,j} = 0$). A similar analysis was carried out for a maximum uncertain coefficient of $\Gamma_{i,j} = 24$, which is shown as the green dashed line in the same figure. Correspondingly, the indoor temperature profiles under the two values of $\Gamma_{i,j}$ are illustrated in Fig. 4. Thus, given $\Gamma_{i,j} = 0$, the reference energy profile is very close to the upper envelope, and with the added uncertainty in the electricity price, it moves toward the lower envelope. We compare the simulation results with a deterministic problem as a benchmark, in which

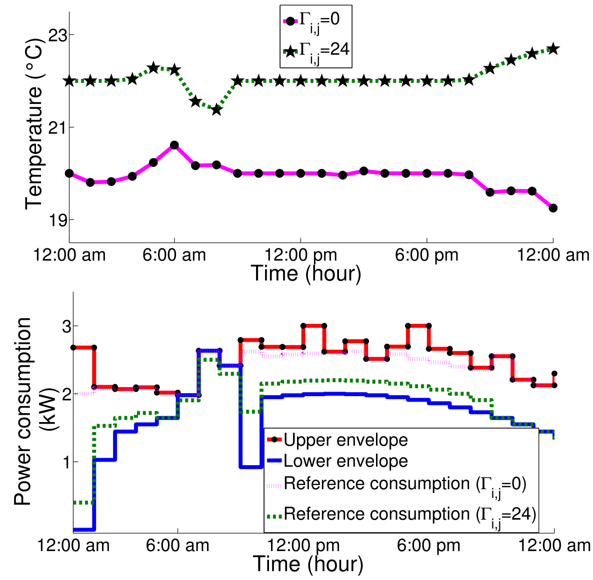

Fig. 4. Comparison of temperature, power consumption, and up/down reserve for $\Gamma_{i,j} = 0$ and $\Gamma_{i,j} = 24$, given the desired temperature of 21 °C.

TABLE I
COMPARISON OF DAILY ENERGY PAYMENT (¢) VERSUS ($\Gamma_{i,j}$)

$\Gamma_{i,j}$	Case I deterministic	Case I robust	Case II deterministic	Case II robust
0	44.15	44.15	44.15	44.15
4	66.53	66.37	64.89	65.15
8	88.65	87.32	85.09	86.44
12	110.13	105.85	105.63	105.85
16	130.64	121.67	126.23	121.36
20	149.82	136.03	146.75	134.76
24	167.31	146.02	167.31	146.02

the electricity price is not a source of uncertainty. Two cases are studied as follows:

Case I: This case is for a comparison of deterministic and robust solutions under the worst realization of price uncertainty. Daily energy payment is calculated for deterministic and robust solutions under different values of $\Gamma_{i,j}$, and the results are summarized in Table I. The results indicate that the robust scheme lessens the energy payment under the worst realizations of the price uncertainty.

Case II: We compare the two solutions with respect to 200 electricity price realizations that were randomly generated based on the $\Gamma_{i,j}$ coefficient. Table I shows the average energy payment under different realizations of real-time price. Based on the results, the robust solution can lead to less energy payment for values of the $\Gamma_{i,j}$ coefficient greater than 12.

B. ISO and Aggregator Interaction

To solve the problem in layer 2, the aggregated up/down-reserve capacity ($\overline{\mathbf{V}}_{j,h,n}, \underline{\mathbf{V}}_{j,h,n}$) that each aggregator can offer to the ISO is obtained from step 1 for three different profiles of reserve rewards ($N = 3$). Figs. 5 and 6 demonstrate the hourly up/down reserve the ISO buys in the DAM as the green line,

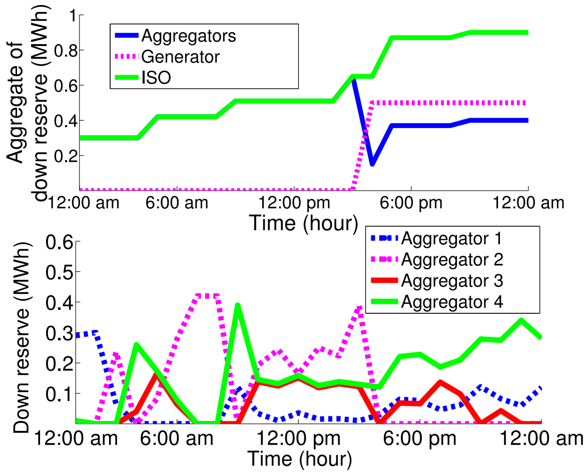

Fig. 5. Hourly down reserve the ISO needs and purchases in the DAM from the generator and aggregators.

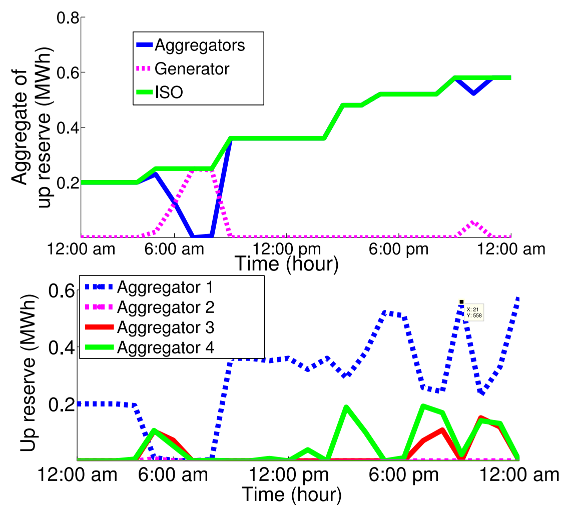

Fig. 6. Hourly up reserve the ISO needs and purchases in the DAM from the generator and aggregators.

which can be provided by either the aggregators or the generator. Traditionally, the generator capacity is used for FRS. However, frequently changing the generator power output may shorten its lifetime and also produces excessive CO_2 emissions. Thus, we assume the cost of procuring reserve by the generator is much higher compared with the aggregators, which leads to a high proportion of the ISO's reserve to be met by the aggregators. The reserve capacity that each aggregator could sell in the wholesale electricity market is shown in the same figures. This capacity is calculated based on the aggregator's costs for purchasing up/down reserves from the users $\{\overline{f}_c(\overline{V}_{j,h}), \underline{f}_c(\underline{V}_{j,h})\}$, which depends on the reward that an aggregator offers to them.

C. Demand-Side Reserve Allocation

Up to now, a contract between the ISO and each aggregator is set, which quantifies how much reserve an aggregator could sell in the market. If an aggregator could successfully sell all the end users' flexibilities in the market, it makes a contract

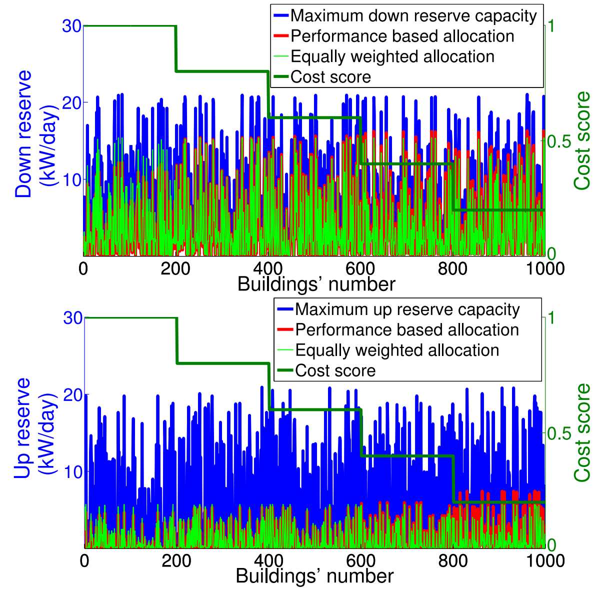

Fig. 7. Comparison of performance-based reserve allocation with the equally weighted one in kW/day.

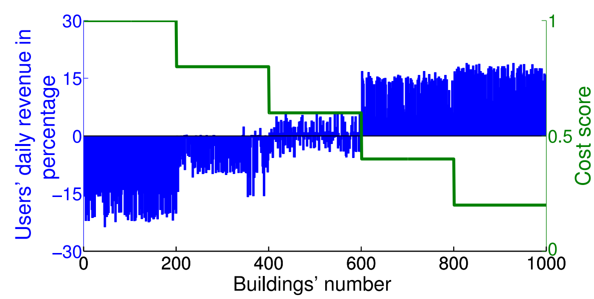

Fig. 8. Performance-based users' daily revenue in percentage of equally weighted reserve allocation.

with each participant building and equally shares the reserve and benefits among them. Otherwise, the reserve capacity that an aggregator requires to buy from end users is less than their submitted reserve in step 1, and thus, the aggregator performs an allocation. In this part, the performance-based allocation of reserve among users is compared with an equally weighted allocation. We assume a range of FRS performance scores for the 1000 participant buildings, as shown in Fig. 7. The users with a lower cost score can sell a higher proportion of their submitted reserve to the aggregator and they may gain more revenues in percentage of equally weighted allocation, which is depicted in Fig. 8. According to the proposed model, the aggregator has a preference to have its contract with more reliable building owners, since both the utility's FRS and aggregators' profits will be less at risk.

VI. Conclusion

Renewable energy sources are integrated into the smart grid. However, the major drawback of renewables is the intermittency of their output, which can deteriorate the functionality of the grid. To this end, demand-side flexibility has been studied to help mitigate the aforementioned drawbacks with a particular focus on a hierarchical DR market. A three-step algorithm has been applied to the proposed model. Specifically, in step 1, the building owners offer reserve capacity to the aggregator with the right to be remunerated for the capacity offered. Meanwhile, a robust optimization model has been proposed in this step to account for the uncertainty of the electricity price. In step 2, the aggregator attempts to sell its reserve capacities to the ISO in a trading market. To model the interaction between the ISO and aggregators, a bilevel optimization problem has been solved with the aim of minimizing the cost of the ISO and maximizing the aggregators' benefits in the DAM. In step 3, the aggregator allocates the reserve among users based on their maximum flexibilities and the sold reserve in step 2. We compared the performance-based allocation of reserve with the equally weighted one and observed that a higher percentage of daily revenue can be obtained by building owners with a higher performance as a result of their more reliable past performance.

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