

# Real-time Market-based Coordination Mechanism for Transmission and Distribution Networks

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**Abstract**—Distributed energy resources (DERs) such as renewable generation, energy storage and flexible loads have been acknowledged as the key drivers to transform the passive distribution networks into proactive players in the electricity market. In order to reduce the communication and computational complexity of integrating the huge number of DERs into the power system operation, the distribution system operator (DSO) is considered as the central controller to take charge of the dispatch of all the DERs within a distribution network. Through the exchange of boundary power flows and control signals, the independent system operator (ISO) coordinates all the DSOs to achieve certain social objective. In this paper, we study the interactions between the ISO and all the DSOs in both the day-ahead market and the real-time balancing market. Particularly, we identify that the locational marginal price (LMP) based real-time market is unfair and discourages the integration of DERs. To achieve a win-win solution for both the ISO and the DSOs, we propose a real-time coordination mechanism to determine the power dispatch and the corresponding charge/payments via a Nash bargaining problem. Numerical results show that our proposed mechanism guarantees that the ISO and all the DSOs can cooperatively maximize the social welfare and share the benefits fairly due to the cooperation.

## I. INTRODUCTION

In order to increase the penetration of renewable energy and improve the energy efficiency, distributed energy resources (DERs) have been greatly promoted and integrated into the power systems. DERs comprise of distributed generators, energy storage, and flexible loads, which are connected to the distribution network. Traditionally, the distribution networks are passive with given demands and the ISO is only in charge of the economic dispatch of the generators at the transmission-network level. However, the direct control and dispatch of DERs will introduce both high communication and computational complexities to the ISO. Thus, to facilitate the participation of DERs in the power system operation and market clearing process, an aggregator or distribution system operator (DSO) is essentially needed to take the role of aggregating all the DERs and creating an electricity market at the distribution level [1]. However, without system-wide cooperation, each DSO will clear its market and determine the dispatch of DERs only based on its local information to optimize its local objective, which may conflict with the objectives of other DSOs in the transmission network operation. Therefore, to improve the efficiency and reliability of the entire system, it is of great importance for the ISO to coordinate all the DSOs via the exchange of the boundary power flows and prices information as shown in Fig. 1.

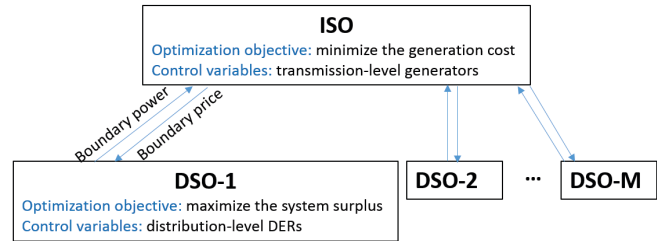


Fig. 1. The interactions between the ISO and the DSOs

Two streams of literature have been carried out to study the coordination mechanism between the ISO and all the DSOs. The first stream focuses on designing the decentralized algorithm for the power dispatch of the cooperative ISO and DSOs. [2] proposes an iterative algorithm that decomposes the centralized system-wide operation problem into a transmission sub-problem and several distribution sub-problems based on the DC power flow model. The sub-problems can be iteratively solved in a distributed manner with an exchange of the boundary information. [3] extends [2] by considering the AC power flow model to improve the voltage management and security operation of the integrated system. [4] proposes a decomposed algorithm based on proximal message passing (PMP), and with extra consideration on the effective provision of DER reserves to further guarantee the power system integrity. [5] extends [4] by proposing a correction filter to improve the convergence. However, this stream of works does not consider the association between the forward market and the balancing market. In the power system operation, the boundary load injections and clearing prices are firstly determined in the day-ahead economic dispatch. After the realization of the real-time renewable generation and demand, the DSOs may deviate from their day-ahead load injection decisions at the distribution substations by controlling their DERs to improve the customers' utilities or maintain the stability of the distribution network. However, the mismatch of load injections between the day-ahead and real-time will lead to changes of the ISO's operation in the transmission network. [6] studies the real-time operation of the distribution grids. It assumes the assigned power and LMP at the substation are constant on the day-ahead basis. The DSO has to balance the benefits obtained from the real-time mismatch and the penalty introduced by the deviation by adding another term in the objective to penalize violations in the assigned power. However, the penalty term is heuristically chosen as an absolute linear function of the mismatch. [7]

completes the story raised in [6] by considering the DSOs firstly submit bids to the ISO, and then do the local scheduling based on the published LMP from the ISO. However, the interaction between transmission and distribution is still not fully captured since the decision of the DSO has no influence back on the ISO's operation.

Although the LMP-based mechanism [8] can exactly reflect the true value of energy from different transmission-level generators and DERs, the real-time market is intricately an adjustment market of the day-ahead market. Since the ISO and all the DSOs expect to positively benefit from the cooperation of the power system operation, each agent has to charge or pay to share the improved social welfare from the real-time adjustment, which includes both the changes of DSOs' customer utilities and the ISO's operation costs. Due to the uncertain realization of the renewable generation and demands of all the distribution networks, the real-time LMPs are not equally observed or estimated by all the DSOs in the day-ahead. Consequently, on the one hand, the DSOs with poor forecast ability for clearing prices would be discouraged to take active operation. On the other hand, the strategic DSOs with large loads or great knowledge of the power grid could make profits through the load mismatch, which may affect the power system stability. Thus, it is of great importance to design a coordination mechanism for the ISO and the DSOs so that they can operate cooperatively to maximize the net-benefit of the whole power system, and share the benefit fairly. In this paper, we focus on the design of such a coordination mechanism.

Our contributions can be summarized as follows: 1) We propose a market-based coordination mechanism for real-time power dispatch and pricing among the cooperative ISO and DSOs based on the Nash bargaining framework. 2) The bargaining-based mechanism guarantees that the ISO and each DSO can positively benefit from the cooperation, and the benefit is fairly allocated among them. Therefore, each DSO is encouraged to participate in the cooperation by actively controlling its DERs. 3) We propose decentralized and efficient algorithms to cope with the interactions and coordination between the ISO and the DSOs since it is difficult to coordinate them in a centralized manner due to privacy issue, large diversity and high computational complexity.

## II. DAY-AHEAD MARKET

We focus on the economic dispatch problem in the day-ahead market. Let  $\mathcal{T} = \{1, 2, \dots, T\}$  be the set of time slots of one day. Denote by  $\mathcal{N}$  the set of the transmission-level buses. Furthermore, let  $\mathcal{N}^g$  and  $\mathcal{N}^{fg}$  denote the set of buses connected to online generators and fast start generators, respectively. The online generators have already been started during the day-ahead economic dispatch while the fast-start generators are off at the beginning of the time horizon and can be turned on during the real-time operations.  $G_n(\cdot)$  and  $G_n^f(\cdot)$  denote the convex generation cost of the online generators and fast-start generators, respectively. Let  $\mathcal{L}$  denote the set of transmission lines.

Let  $\mathcal{M}$  be the set of distribution networks,  $m \in \mathcal{M}$ . Denote by  $\mathcal{N}^m$  the set of buses in distribution network  $m$ . For the adjustable flexible load (e.g., HVAC) with a power consumption of  $P_i^m(t)$  in distribution  $m \in \mathcal{M}$  at bus  $i \in \mathcal{N}^m$  at time  $t \in \mathcal{T}$ , let  $U_i^m(P_i^m(t))$  denote its utility, where  $U_i^m(\cdot)$  is assumed to be a profile-dependent concave function whose maximum is achieved when  $P_i^m(t) = \tilde{P}_i^m(t)$  for  $t \in \mathcal{T}$ . For the deferrable load (e.g., electric vehicles) with a total power consumption of  $\sum_{t \in \mathcal{T}} P_i^m(t)$  in the distribution network  $m$  at bus  $i$ ,  $U_i^m(\sum_{t \in \mathcal{T}} P_i^m(t))$  denote its utility over the time horizon. For simplicity of presentation, we denote the utility function as  $U_i^m(\mathbf{P}_i^m)$ , where  $\mathbf{P}_i^m = \{P_i^m(t)\}_{t \in \mathcal{T}}$ .

In the day-ahead market, the ISO and the DSOs cooperatively implement the economic dispatch to maximize the social welfare by solving the optimization problem:

$$\max_{\substack{g_n(t) \in \mathcal{P}^g, \\ P_i^m(t) \in \mathcal{P}_i^m}} \sum_{t=1}^T \left[ \sum_{m \in \mathcal{M}} \sum_{i \in \mathcal{N}^m} U_i^m(\mathbf{P}_i^m) - \sum_{n \in \mathcal{N}^g} G_n(g_n(t)) \right]$$

where the decision variable  $g_n(t)$  is the generation output at bus  $n$ .  $\mathcal{P}^g$  and  $\mathcal{P}_i^m$  denote the feasible region of the transmission-level generators and distribution-level flexible loads, respectively.  $\mathcal{P}^g$  and  $\mathcal{P}_i^m$  are constrained by the power flow equations in the transmission and distribution networks, which will be specified in the following.

Following the decomposition procedures in [2], the social welfare maximization problem can be decoupled into the economic dispatch sub-problems for the ISO and the DSOs. The optimal solution of the social welfare problem can be achieved by iteratively solving these sub-problems.

Given the load injections  $d^m(t)$  from DSO- $m$  at time  $t$ , the ISO solves its economic dispatch problem:

$$\begin{aligned} \min_{g_n(t)} \quad & \sum_{t=1}^T \sum_{n \in \mathcal{N}^g} G_n(g_n(t)) \\ \text{s.t.} \quad & \sum_{n \in \mathcal{N}^g} g_n(t) = \sum_{n \in \mathcal{N}} d_n(t), \forall t \in \mathcal{T} \\ & -c_l \leq \sum_{n \in \mathcal{N}} S_{n,l}(g_n(t) - d_n(t)) \leq c_l, \forall (l \in \mathcal{L}, t \in \mathcal{T}) \\ & 0 \leq g_n(t) \leq g_n^{max}, \forall (n \in \mathcal{N}^g, t \in \mathcal{T}) \end{aligned} \quad (1)$$

Constraints (1) are the power balance and  $d_n(t)$  is the load injection at bus  $n$  at time  $t$ . Note that the load injection  $d^m(t)$  at the substation should be equal to the corresponding  $d_n(t)$  in the transmission network. (2) are the transmission line capacity constraints, where  $c_l > 0$  is the transmission line capacity and  $S_{n,l}$  is the shift factor of line  $l$  with respect to bus  $n$ . (3) are the generator capacity constraints, where  $g_n^{max}$  is the maximum generation output. Let  $\lambda(t)$  and  $(\mu_{l,1}(t), \mu_{l,2}(t))$  be the optimal duals associated with constraints (1) and (2), respectively. We can determine  $\pi_n(t)$ , LMP of bus  $n$  at time  $t$  by combining these optimal dual variables [9]:

$$\pi_n(t) = \lambda(t) + \sum_{l \in \mathcal{L}} S_{n,l}(\mu_{l,1}(t) - \mu_{l,2}(t))$$

Given the LMP  $\pi_0^m(t)$  at the boundary bus from the ISO, the economic dispatch problem for each DSO-m is as follows:

$$\begin{aligned} \max_{\mathbf{x}} \quad & \sum_{t=1}^T \left[ \sum_{i \in \mathcal{N}^m} U_i^m(\mathbf{P}_i^m) - \pi_0^m(t) d^m(t) \right] \\ \text{s.t.} \quad & P_{i0}^m = r_{i0}^m l_{i0}^m - d^m + \sum_k P_{0k}^m, \forall i \end{aligned} \quad (4)$$

$$Q_{i0}^m = x_{i0}^m l_{i0}^m - q_0^m + \sum_k Q_{0k}^m, \forall i \quad (5)$$

$$P_{ij}^m = r_{ij}^m l_{ij}^m - p_j^m + P_j^m + \sum_k P_{jk}^m, \forall (i, j \neq 0) \quad (6)$$

$$Q_{ij}^m = x_{ij}^m l_{ij}^m - q_j^m + Q_j^m + \sum_k Q_{jk}^m, \forall (i, j \neq 0) \quad (7)$$

$$v_j^m = v_i^m + (r_{ij}^m l_{ij}^m + x_{ij}^m l_{ij}^m) - 2(P_{ij}^m r_{ij}^m + Q_{ij}^m x_{ij}^m), \forall (i, j) \quad (8)$$

$$v_i^m = \frac{1}{l_{ij}^m} (P_{ij}^m l_{ij}^m + Q_{ij}^m l_{ij}^m), \forall (i, j) \quad (9)$$

$$\underline{v}_i^m \leq v_i^m(t) \leq \bar{v}_i^m, \forall (i, t) \quad (10)$$

$$0 \leq p_i^m(t) \leq \tilde{p}_i^m(t), \forall (i, t) \quad (11)$$

$$\underline{q}_i^m \leq q_i^m(t) \leq \bar{q}_i^m, \forall (i, t) \quad (12)$$

$$Q_i^m(t) = P_i^m(t) \tan(\phi_i^m), \forall (i, t) \quad (13)$$

$$0 \leq P_i^m(t) \leq \bar{P}_i^m, \forall (i, t) \quad (14)$$

$$\sum_{\substack{t_i^m \\ t_i^m}} P_i^m(t) = \widetilde{PC}_i^m, \forall i \quad (15)$$

where the tree is rooted at the substation bus indexed by  $i = 0$ . The set of decision variables  $\mathbf{x}$  is the power dispatch of the distribution network, including: the active and reactive load at bus  $i$ , i.e.  $P_i^m(t)$  and  $Q_i^m(t)$ ; the active and reactive power outputs of renewable generation such as wind turbine and solar panel, i.e.  $p_j^m(t)$  and  $q_j^m(t)$ ; the active and reactive power injection from the transmission grid at the substation  $d^m(t)$  and  $q_0^m(t)$ . (4)-(9) are the branch flow model [10], where the time indices are omitted for simplicity.  $S_{ij}^m(t) = P_{ij}^m(t) + jQ_{ij}^m(t)$  and  $z_{ij}^m = r_{ij}^m + jx_{ij}^m$  are the complex power flow and line impedance of line  $(i, j)$ , respectively.  $v_i^m(t)$  and  $l_{ij}^m(t)$  denote the squared voltage magnitude on bus  $i$  and the squared current magnitude of line  $(i, j)$ , respectively. (10) are the voltage magnitude constraints. (11) and (12) are the generation capacity constraints, where  $\tilde{p}_i^m(t)$  is the maximum renewable generation output. (13)-(15) are the load specific constraints, where  $\tan(\phi_i^m)$  is the power factor of specific load.  $\widetilde{PC}_i^m$  is the required cumulative load.

Leveraging the second-order cone program (SOCP) relaxation technique for the distribution network with a tree topology [10], we further replace constraints (9) with

$$v_i^m(t) \geq \frac{1}{l_{ij}^m(t)} (P_{ij}^m(t)^2 + Q_{ij}^m(t)^2), \forall (i, t) \quad (16)$$

Based on the above relaxation, the economic dispatch problem of DSO-m is a convex problem and can be solved efficiently

and optimally. DSO-m will send the optimal solution of the load injection at the boundary bus  $d^m(t)$  to the ISO for the next iteration.

Starting with an initial guess of the boundary load injections, the ISO and the DSOs solve their corresponding economic dispatch iteratively by exchanging the boundary load injection  $d^m(t)$  and boundary bus LMP  $\pi_0^m(t)$ . It has been proved that the original social welfare maximization problem can be solved optimally by this decentralized algorithm with convergence guarantee [2]. After the market clearing process, the optimal day-ahead dispatch and LMPs  $\hat{d}^m(t)$ ,  $\hat{\pi}_0^m(t)$  and  $\hat{g}_n(t)$  are determined.

### III. REAL-TIME MARKET

In this section, we focus on the economic dispatch at each decision time slot  $t_0 \in \mathcal{T}$  in the real-time market. For  $t = 1$  to  $t_0 - 1$ , all the dispatches are determined; for  $t = t_0$ , new information are given such as preferred load  $\tilde{P}_i^m(t_0)$ , preferred cumulative load  $\widetilde{PC}_i^m$  and maximum renewable generation  $\tilde{p}_i^m(t_0)$ ; for  $t = t_0 + 1$  to  $T$ , preferred load and renewable generation are still unknown, so we use the day-ahead predicted values. Below we present three economic dispatch models, which include the noncooperative model, the LMP-based cooperative model, and the NBP-based cooperative model.

#### A. Noncooperative Benchmark

According to current practice, the DSOs cannot violate the committed boundary power injections from the day-ahead market. Each DSO can only manipulate the DERs within its own distribution network since it has no cooperation with other operators. Because of the uncertain realization of renewable generation and preferred demands, the DSO suffers from the risk of wasting surplus of the renewable generation or degrading the utilities of the flexible loads.

In this uncooperative case, the economic dispatch of the ISO just follows the day-ahead decisions, while the DSO-m solves the following dispatch problem at each  $t_0$  where  $d^m(t)$  is restricted to be  $\hat{d}^m(t)$ :

$$\begin{aligned} \max_{\mathbf{x} \setminus \{d^m(t)\}} \quad & \sum_{t=t_0}^T \sum_{i \in \mathcal{N}^m} U_i^m(\mathbf{P}_i^m), \\ \text{s.t.} \quad & \text{Constraints (4)-(8), (10)-(16).} \end{aligned}$$

Although we obtain the optimal dispatch for the total remaining time horizon, only the decisions for the current time slot, which is denoted by  $\hat{P}_i^m(t_0)$ , are used for the dispatch of time slot  $t_0$ . Let  $\mathcal{U}_{Non}^m(t_0)$  denote the optimal value of the above optimization problem. We further define the generation costs and customer utilities over the whole time horizon in the noncooperative case as  $\mathcal{G}_{Non} = \sum_{t=1}^T \sum_{n \in \mathcal{N}^g} G_n(\hat{g}_n(t))$  and  $\mathcal{U}_{Non}^m(t_0) = \sum_{t=1}^{t_0-1} \sum_{i \in \mathcal{N}^m} U_i^m(\hat{\mathbf{P}}_i^m) + \mathcal{U}_{Non}^m(t_0)$ , respectively.

### B. LMP-based Cooperation

If the ISO and the DSOs determine the real-time economic dispatch and corresponding charge/payments for the mismatch of the boundary load injections based on the real-time LMP, additional benefits for the whole system (e.g., increasing customer utilities or decreasing operational costs) can be achieved.

Note that additional costs are introduced to the ISO's dispatch problem in the real-time market due to the load deviations at the substations. Particularly, if a generator is asked to reduce the real-time output, i.e.  $g_n < \hat{g}_n$ , it should be compensated for the *LOC* by the ISO. We denote by  $\hat{\pi}_n$  the day-ahead clearing price at bus  $n$ . *LOC* is defined as the difference between the benefit that a generator at bus  $n$  should have earned  $\hat{R} = \hat{\pi}_n \hat{g}_n - G_n(\hat{g}_n)$  and the benefit that it will earn without compensation  $R = \hat{\pi}_n g_n - G_n(g_n)$ , namely,

$$LOC = \hat{R} - R = \hat{\pi}_n(\hat{g}_n - g_n) - G_n(\hat{g}_n) + G_n(g_n).$$

By the compensation of *LOC*, the generators are guaranteed to obtain the same benefit as the case without load deviations from the DSOs. Thus, we can define the real-time generation cost as:

$$G'_n(g_n) = \begin{cases} G_n(g_n) + LOC & \text{if } 0 \leq g_n < \hat{g}_n, \\ G_n(g_n) & \text{if } \hat{g}_n \leq g_n \leq g_n^{max}. \end{cases}$$

In other cases, with increasing real-time load, some fast-start generators need to be committed when online generators cannot meet the demand because of maximum output limits and transmission line capacities. Basically, apart from the set of already committed online generators, we further include another set of fast-start generators with higher costs. Thus, we can further define the real-time generation cost  $\sum_{n \in \mathcal{N}^g} G'_n(g_n) + \sum_{n \in \mathcal{N}^{fg}} G_n^f(g_n)$ , which accounts for both the *LOC* and the fast-start generation cost.

At each time slot  $t_0$ , the ISO and the DSOs cooperatively implement the economic dispatch based on the social welfare maximization problem given by:

$$\max_{\substack{g_n(t) \in \mathcal{P}^g, \\ P_i^m(t) \in \mathcal{P}_i^m}} \sum_{t=t_0}^T \left[ \sum_{m \in \mathcal{M}} U_i^m(\mathbf{P}_i^m) - \sum_{n \in \mathcal{N}^g} G'_n(g_n(t)) - \sum_{n \in \mathcal{N}^{fg}} G_n^f(g_n(t)) \right]$$

which is similar to the social problem in the day-ahead market and can be solved efficiently with the same decentralized algorithm. Also, only the decision for the current time slot denoted by  $g_n^*(t_0)$ ,  $P_i^{m*}(t_0)$  and  $d^{m*}(t_0)$  will be used to do the real dispatch, along with the by-product  $\pi_0^{m*}(t_0)$ . We use  $\mathcal{G}_{Co}^*(t_0)$  and  $\mathcal{U}_{Co}^{m*}(t_0)$  to denote the optimal generation costs and customer utilities over the remaining time horizon respectively resulting from the sub-problems. We further define the generation costs and customer utilities over the whole time horizon in the LMP-based cooperation as  $\mathcal{G}_{Co}(t_0) = \sum_{t=1}^{t_0-1} [\sum_{n \in \mathcal{N}^g} G'_n(g_n^*(t)) + \sum_{n \in \mathcal{N}^{fg}} G_n^f(g_n^*(t))] + \mathcal{G}_{Co}^*(t_0)$  and  $\mathcal{U}_{Co}^m(t_0) = \sum_{t=1}^{t_0-1} \sum_{i \in \mathcal{N}^m} U_i^m(\mathbf{P}_i^{m*}) + \mathcal{U}_{Co}^{m*}(t_0)$ , respectively.

The real-time payment based on LMP is determined in this real-time economic dispatch problem. The payment for DSO- $m$  is calculated by multiplying the load injection deviation with

the real-time boundary LMP, i.e.,  $\pi_0^{m*}(t)(d^{m*}(t) - \hat{d}^m(t))$ . The charge of the ISO is the summation of all the DSOs' payments. However, this payment scheme can not take care of the fair allocation of the system benefit in terms of the improved social welfare. As a result, the DSOs lack incentives to participate in the real-time cooperation under the LMP-based payment scheme. Towards this end, we propose the NBP-based cooperation mechanism in the following subsection.

### C. NBP-based Cooperation

We first define the net-benefits (i.e., the additional time-average benefits compared with the noncooperative benchmark since the net-benefit is time-correlated) of the ISO and the DSOs at time slot  $t_0$  as follows:

$$B^0(t_0) = c(t_0) - \frac{1}{T}(\mathcal{G} - \mathcal{G}_{Non}),$$

$$B^m(t_0) = \frac{1}{T}(\mathcal{U}^m - \mathcal{U}_{Non}^m(t_0)) - b^m(t_0),$$

where  $\mathcal{G} = \sum_{t=1}^T [\sum_{n \in \mathcal{N}^g} G'_n(g_n(t)) + \sum_{n \in \mathcal{N}^{fg}} G_n^f(g_n(t))]$  and  $\mathcal{U}^m = \sum_{t=1}^T \sum_{i \in \mathcal{N}^m} U_i^m(\mathbf{P}_i^m)$ .

We model the interactions between the ISO and the DSOs as a Nash bargaining problem (NBP) as below:

$$\begin{aligned} \text{NBP:} \quad & \max_{\substack{g_n(t), \mathbf{x}, \\ c(t_0), b^m(t_0)}} B^0(t_0) \prod_{m \in \mathcal{M}} B^m(t_0) \\ \text{s.t.} \quad & c(t_0) \leq \sum_{m \in \mathcal{M}} b^m(t_0), \\ & \text{Constraints (1)-(3), (4)-(8), (10)-(16).} \end{aligned}$$

There are two groups of decision variables in the NBP: 1)  $g_n(t)$  and  $\mathbf{x}$  regarding the power dispatch; and 2) the ISO's charge  $c(t_0)$  and the corresponding DSOs' payments  $b^m(t_0)$ .

The bargaining problem represents that the ISO and all the DSOs collaborate to maximize the product of their net-benefits, from which they can simultaneously achieve satisfactory net-benefits and also reach an agreement on their charge and payments. In order to solve the non-convex NBP problem, we decompose the problem into two consecutive convex optimization problems, i.e. a social welfare maximization problem and a benefit allocation problem. Thus, we propose a two-step solution method as follows.

**Step-I:** Due to the fact that the ISO's charge should balance the DSOs' total payments exactly at the equilibrium of the bargaining problem, NBP can be associated with a social welfare maximization problem as follows:

$$\begin{aligned} \max_{g_n(t), \mathbf{x}} \quad & \frac{1}{T} \left[ \sum_{m \in \mathcal{M}} (\mathcal{U}^m - \mathcal{U}_{Non}^m(t_0)) - (\mathcal{G} - \mathcal{G}_{Non}) \right] \\ \text{s.t.} \quad & \text{Constraints (1)-(3), (4)-(8), (10)-(16).} \end{aligned}$$

It can be proved that the optimal decisions regarding the power dispatch for the original bargaining problem, also suffice to be optimal with respect to the social problem [11]. By discarding the determined terms in the objective, the Step 1 problem happens to be further simplified into exactly the same social problem in the cooperative model based on LMP payment.

**Step-II:** By substituting the optimal power dispatch  $g_n^*(t)$  and  $\mathbf{x}^*$  obtained from **Step-I** into the NBP, we can continue to solve the original Nash bargaining problem shown below to determine the ISO's optimal charge  $c^*(t_0)$  and all the DSOs' optimal payments  $b^{m*}(t_0)$ .

$$\begin{aligned} \max_{c(t_0), b^m(t_0)} \quad & [c(t_0) - \frac{1}{T}(\mathcal{G}_{Co}(t_0) - \mathcal{G}_{Non}(t_0))] \\ \text{s.t.} \quad & \prod_{m \in \mathcal{M}} [\frac{1}{T}(\mathcal{U}_{Co}^m(t_0) - \mathcal{U}_{Non}^m(t_0)) - b^m(t_0)] \\ & c(t_0) \leq \sum_{m \in \mathcal{M}} b^m(t_0), \end{aligned}$$

Under the NBP-based cooperation, not only the optimal social welfare can be achieved with full consideration of the coordination between transmission and distribution networks, but also the net-benefit from the cooperation is allocated fairly between the ISO and the DSOs. Therefore, the payment scheme based on the bargaining solution motivates the DSOs to actively participate in the real-time cooperation.

#### IV. CASE STUDY

In our simulation, the time duration of each time slot is set to be one hour and  $T = 24$ . We investigate the optimality and fairness of our coordination mechanism on the modified IEEE 14-bus system connected to four distribution systems at transmission buses #6, #9, #13 and #14 as shown in Fig. 2 [3].  $G_1, G_2, G_3$  and  $G_4$  are the online generators, while  $G_5$  is the fast-start generator with higher generation cost. The generation cost is assumed to be  $G_n(g_n(t)) = ag_n(t)^2 + bg_n(t)$ . The generator parameters are given in Table I.

All the distribution networks are 6-bus systems, sharing the same topology and line data in p.u. given in Fig. 3 [12]. The radial network is rooted at bus #0 with the voltage  $v_0^m = 1p.u.$ . The voltage magnitude limits are  $\underline{v}_i^m = 0.9$  and  $\bar{v}_i^m = 1.05$ . We assume  $D1$  and  $D3$  are residential networks equipped with wind turbines at bus #1 sharing the same renewable generation and preferred load profiles, while  $D2$  and  $D4$  are commercial networks equipped with solar panels at bus #1. The reactive power output limits are determined by the active power limit, i.e.,  $\underline{q}_i^m = -\tilde{p}_i^m(t)$  and  $\bar{q}_i^m = \tilde{p}_i^m(t)$ . The prediction and realization of renewable generation profiles are from *California Independent System Operator* [13] and are shown in Fig. 4. For aggregated electric vehicles at bus #2, the charging power limits are  $\underline{P}_2^m = 0$  and  $\bar{P}_2^m = 20MW$  without reactive load injection. For the two residential networks, the charging time horizon is from 6 p.m. to 8 a.m. with a total energy requirement of  $81.265MWh$ . For the two commercial networks, the charging time horizon is from 8 a.m. to 6 p.m with a total energy requirement of  $81.135MWh$ . The adjustable flexible loads at bus #3, #4, #5 are assumed to have the same profiles from *OpenEI* [14] as shown in Fig. 5. The power factor is set to be identical, namely,  $\tan(\phi_i^m) = 1/3, i = 3, 4, 5$ . The cumulative flexible loads are restricted to be consistent with the preferred values. The utility functions are in the form of  $U_i^m(\mathbf{P}_i^m) = -10\mathbf{P}_i^{m2} + 20\mathbf{P}_i^m$ .

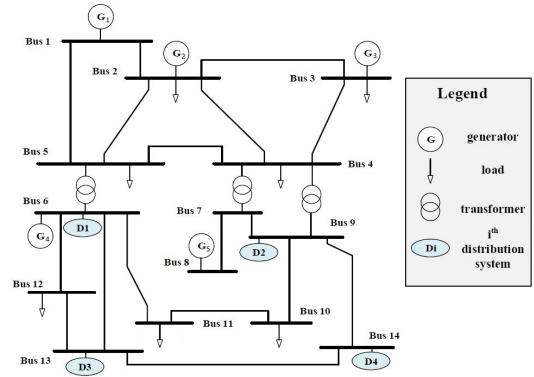


Fig. 2. Topology of the 14-bus test system

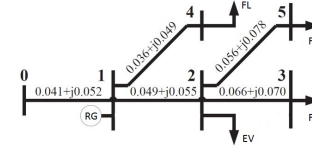


Fig. 3. Topology of the 6-bus distribution system

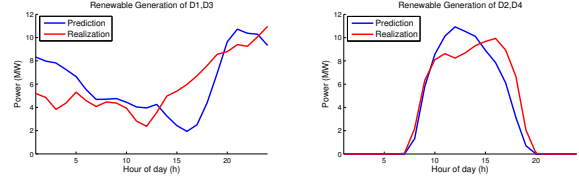


Fig. 4. Renewable generation profiles

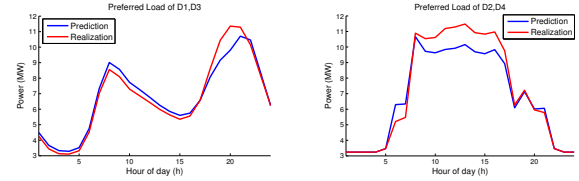


Fig. 5. Preferred flexible load profiles

We simulate the day-ahead market model and the real-time market models including the noncooperative benchmark and the cooperative models. Fig. 6 shows the optimal social welfare which is the summation of the customer utilities of all the distribution networks and the negative generation costs over the whole time horizon at all decision time slots for both noncooperative and cooperative real-time models. It can be seen that the social welfare for the cooperative models is always higher than that of the noncooperative benchmark. The cooperation can improve the real-time coordination between the ISO and the DSOs to achieve the optimal social welfare. Fig. 7 shows the optimal customer utilities of each distribution network and the optimal generation costs separately. It can be observed that at some time slots, e.g.  $t = 9, 10$  for  $D4$ ,

TABLE I  
GENERATOR PARAMETERS

	$a(\$/MWh^2)$	$b(\$/MWh)$	Capacity(MW)
$G_1$	0.05	20	140
$G_2$	0.2	20	60
$G_3$	0.03	40	50
$G_4$	0.04	40	45
$G_5$	0.5	50	40



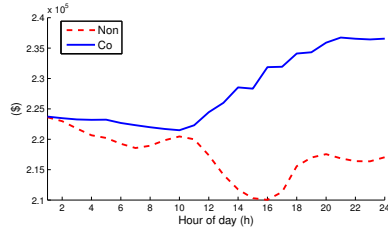


Fig. 6. The social welfare

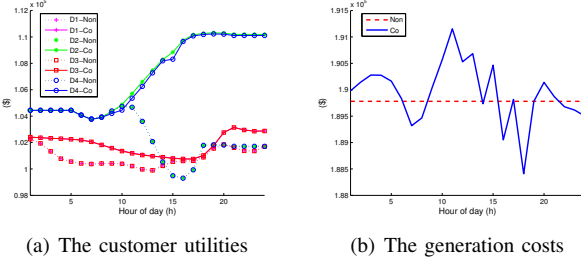


Fig. 7. Customer utilities and generation costs

the optimal overall utilities for the cooperative model are even lower than the noncooperative benchmark. However the resulting social welfares are still higher than the benchmark. It means that the distribution network may sacrifice its own utilities to achieve optimal social welfare of the whole system. Furthermore, after one day's realization, the actual generation costs given at the last decision slot in Fig. 7(b) are lower in the cooperative models.

TABLE II

THE COMPARISON BETWEEN THE NBP APPROACH AND LMP SCHEME

		Charge / Payment	Net-benefit	Fairness Index
LMP	ISO	6496.825	6775.799	0.716
	DSO-1	101.7118	1084.522	
	DSO-2	2355.752	6122.037	
	DSO-3	215.8652	962.5504	
	DSO-4	3823.496	4569.258	
NBP	ISO	2264.569	2543.542	0.817
	DSO-1	-1135.87	2322.107	
	DSO-2	2331.815	6145.974	
	DSO-3	-1144.27	2322.686	
	DSO-4	2212.897	6179.857	

Following the optimal dispatch decisions based on the cooperative models, we further compare our charge and payment scheme based on Nash bargaining solution with the LMP. For the LMP scheme, the payment of the DSO at each time slot is only related to the instantaneous load injection deviation. However, our bargaining scheme takes care of the influence of current dispatch on the social welfare over the whole time horizon. More importantly, after one day's realization, we can get the total charge of ISO  $\sum_{t=1}^T c(t_0)$  and the total payment of DSO-m  $\sum_{t=1}^T b^m(t_0)$ . The actual net-benefit of the ISO and the DSOs are  $B^{0*} = \sum_{t=1}^T c(t_0) - [\mathcal{G}_{Co}(T) - \mathcal{G}_{Non}]$  and  $B^{m*} = [\mathcal{U}_{Co}^m(T) - \mathcal{U}_{Non}^m(T)] - \sum_{t=1}^T b^m(t_0)$ , respectively. We further define the normalized Jain's fairness index [15] as  $J(B^{0*}, \{B^{m*}\}) = \frac{(B^{0*} + \sum_{m=1}^M B^{m*})^2}{(M+1)[(B^{0*})^2 + \sum_{m=1}^M (B^{m*})^2]}$ . It becomes more fair as the increase of  $J$ . Table II shows the total charge and payments, the actual net-benefits and the

fairness index of both the NBP approach and LMP scheme. It can be seen that DSO-1 and DSO-3 which contribute to the social welfare improvement at the cost of decreasing their own utilities are compensated by negative total payments under the bargaining scheme. Our NBP approach improves the fairness index by around 0.1 in comparison with the LMP scheme.

## V. CONCLUSIONS

In this paper, we have investigated the coordination mechanism between the ISO and the DSOs in the real-time electricity market. The interactions between the transmission and distribution networks have been modeled as a NBP, and the optimal economic dispatch and corresponding charge or payments have been derived accordingly. Compared to the noncooperative and existing LMP-based cooperative models, the advantage of our proposed NBP model is that the optimality of the social welfare and the fairness of the payment allocation are guaranteed simultaneously. Therefore, we expect that both the ISO and the DSOs will be more motivated to participate in our proposed cooperation mechanism.

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