

# Optimal Charging Operation of Battery Swapping Stations with QoS Guarantee

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**Abstract**—Electric Vehicle (EV) drivers have an urgent demand for fast battery refueling methods for long distance trip and emergency drive. A well-planned battery swapping station (BSS) network can be a promising solution to offer timely refueling services. However, an inappropriate battery recharging process in the BSS may not only violate the stabilization of the power grid by their large power consumption, but also increase the charging cost from the BSS operators' point of view. In this paper, we aim to obtain the optimal charging policy to minimize the charging cost while ensuring the quality of service (QoS) of the BSS. A novel queueing network model is proposed to capture the operation nature for an individual BSS. Based on practical assumptions, we formulate the charging schedule problem as a stochastic control problem and achieve the optimal charging policy by dynamic programming. Monte Carlo simulation is used to evaluate the performance of different policies for both stationary and non-stationary EV arrival cases. Numerical results show the importance of determining the number of total batteries and charging outlets held in the BSS. Our work gives insight for the future infrastructure planning and operational management of BSS network.

## I. INTRODUCTION

Electric Vehicles (EVs) have been considered as one of the most important ways to reduce greenhouse gas emission and promote renewable energy integration [1]. Motivated by EVs' high driving performance and their potential to save fuel cost, an increasing number of customers begin to adopt EVs. However, two key challenges may hinder EVs' large penetration [2]. The first challenge comes from the high price of EV batteries, which takes up a major proportion of EV capital cost. For instance, due to the high capital cost of batteries, concerns about the lifetime of EV batteries from EV owners may prevent concepts such as smart charging and vehicle-to-grid (V2G) [3] from being implemented temporarily. The other challenge lies in how to fulfill the refueling requests of the increasing EVs. Admittedly, some EV owners can refuel their EVs with an overnight charging, however, deploying battery charger at home is rather expensive and may threaten the security of residential power supply [4]. Furthermore, typical EVs in the current market can only drive for about 100 miles, which makes EV owners always concern whether they can reach their destinations before the battery depletes [8]. This range anxiety problem may not be solved completely without the improvement of battery technology, which is, however, not realistic to be optimistic.

Battery swapping concept, as a relatively novel business model, is a promising solution to eliminate the aforementioned anxiety with its numerous advantages. First, in the battery

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swapping business mode, the expensive batteries are usually owned by the operator of battery swapping station (BSS). The EV owners only need to sign a contract with the BSS for the usage of batteries and pay for the energy they consume [6]. Thus the price of EVs will be more acceptable for ordinary customers and they could be free from the battery maintenance problems. Second, the BSS can finish the battery swapping service within 90 seconds, which is even faster than the traditional gasoline refueling [8]. Besides, it will be much easier for system operators to control the charging procedure while refueling the swapped batteries so that they can reduce the threats resulting from large EV penetration to the electricity grid. Meanwhile, the energy storage capacity offers more chance for BSS to participate in the ancillary service and help better integrate the renewable energy to the main grid.

## A. Related Work

Until now, the BSS network planning and operation is still a new research area and only a few papers discuss about it. In order to satisfy the EV owners with convenient and fast battery swapping service, an effectively operated BSS network must be established, which is essentially a multi-time scale scheduling problem from the optimization point of view. Next, we will give a brief overview of the existing works in this area.

The location and sizing of the BSS are the slow time scale decision variables which will not be modified for several years once implemented. This problem is usually considered from two aspects: 1) minimizing infrastructure cost while ensuring service coverage [2] [10]. In particular, BSS locations and scales are decided based on life circle cost criterion in [10]; 2) maximizing utilities when considering the BSS as a big energy storage device. For instance, reference [9] locates BSSs to satisfy the EV service demand and help integrate renewable energy at the same time by the V2G concept.

After the BSS infrastructure construction, the number of total batteries and charging outlets held in BSS can be adjusted properly according to the practical swapping service demand in middle time scale (such as monthly or even daily). Reference [2] takes both slow and middle time scale decision variables into account. It achieves the optimal BSS location and corresponding holding battery number in terms of minimizing opening and operating cost. However, [2] assumes that each BSS can own charging outlets as many as the total number of batteries, which might be somehow not practical because of the construction cost and cable peak power constraints from the power grid.

Finally, BSS operators make decisions in a real time scale (fast time scale) to achieve the trade off between minimizing the

charging cost in a dynamic traffic environment and ensuring the quality of service (QoS) of BSS, which can be quantized by the blocking rate and mean waiting time for swapping service. For example, the optimal battery charging schedule is formulated as a stochastic control problem in [11]. However, the swapped batteries are considered to be homogeneous without taking the random state of charge (SoC) into consideration. Meanwhile, EVs are assumed to wait for swapping service until they eventually obtain a battery, which may not be accurate for impatient customers.

### B. Our Contributions

We focus on the real time-scale battery charging scheduling and guarantee the QoS of BSS. However, different from the previous works, our paper explicitly takes the random SoC, random arrivals and random charging time into consideration based on a novel queueing network model. Specifically, the contributions include the following three aspects.

1) *Novel Queueing Network Model*: We propose a novel queueing network model for individual BSS, which is an open queue of EVs coupled with a closed queue of batteries. The proposed queueing network model captures the coupling nature of the battery collecting and recharging processes, which can be a general performance analysis framework for BSS.

2) *Formulation in Stationary and Non-stationary EV Arrival Environment*: In order to minimize the battery charging cost with QoS guarantee, we formulate the charging scheduling problem as an infinite horizon Markov decision process (MDP) problem and a finite horizon MDP problem when the EV arrival process is stationary and non-stationary, correspondingly. We obtain the real time battery charging policy by dynamic programming and discuss the performances of different modeling methods in the practical non-stationary EV arrival environment.

3) *Discussion of longer time scale parameters on real time scale decisions*: Based on the optimal charging policies and Monte Carlo simulation results, we analyze the influence of two easily tuned longer time scale variables: the total number of batteries and the number of charging outlets in the BSS. Our discussion gives guidance for the longer time scale parameters estimation and optimization.

The rest of paper is organized as follows: Section II presents individual BSS queueing network model and its real time operational process. Infinite horizon MDP and finite MDP modeling approaches are applied to obtain the optimal policy for battery charging problem in Section III. Section IV gives operational performance analysis based on Monte Carlo simulation results. Finally, Section V concludes our work.

## II. SYSTEM MODEL

### A. BSS Model

We consider a discrete time model for BSS. Time horizon is divided into slots with a fixed length. Let  $k$  denote the time epoch,  $k \in \{0, 1, \dots\}$ . BSS system is modeled as an open queue combined with a closed queue as shown in Fig. 1.

**Open queue of EVs.** EVs coming to the BSS will either enter the swapping server (SS), exchange a fully charged

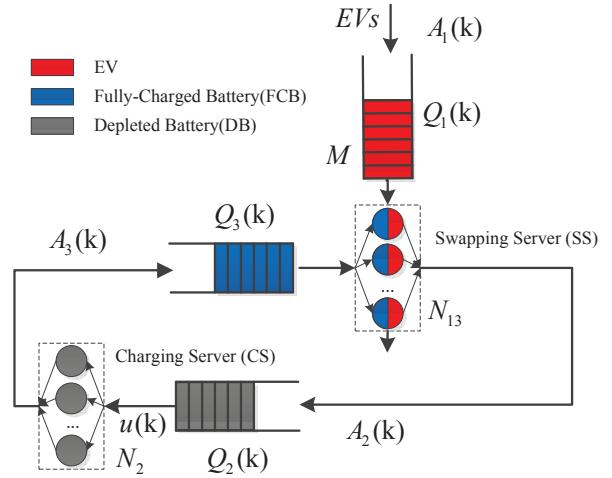


Fig. 1. Queueing network model for individual BSS. The EVs form an open queue and the batteries circulate in a closed queue.

battery (FCB) with its depleted battery (DB) and leave the system or, wait in the parking lots when there are not enough spare swapping islands (i.e., idling servers) or FCBs. It is clear that EVs form an open queue and we use  $Q_1(k)$  to denote the number of EVs waiting in the buffer of the open queue at time  $k$ . If  $Q_1(k)$  reaches its maximum length  $M$ , new EV arrivals will immediately leave for another BSS for service and these lost customers are considered to be blocked.

**Closed queue of batteries.** FCBs and DBs circulate in the FCB queue and DB queue. Let  $N_{13}$  denote the number of swapping islands in SS. Note that each busy swapping island owns an EV from  $Q_1(k)$  and a FCB from  $Q_3(k)$ , which denotes the number of FCBs in the FCB queue. After swapping operation (unloading a DB and then mounting a FCB to the same EV), swapped DBs come out of SS and are delivered to DB buffer  $Q_2(k)$ . It is obvious that the summation of DBs and FCBs, which can be denoted by  $B$ , in the BSS based on the above operation keeps constant. Thus, the queue of batteries is closed. Meanwhile, it is rational to assume that the capacities of DB buffer and FCB buffer are both not less than  $B$ , thus the notorious deadlock and blocking problems will not exist in the closed queue.

The battery swapping operation in SS can be completed quickly within  $D_s$  seconds, which is a constant in practice (e.g.,  $D_s$  can be 90s for Tesla's Model S [8]). However, it will take a relatively longer time  $D_c$  to recharge a DB (e.g., Tesla's Model S needs at least 30 minutes for recharging [8]). These two servers work in different time scales, which incurs the difficulties to maintain the battery flow in the closed queue. In order to simplify the system state evolution process, we assume the system operates following the time conventions in Fig. 2. **Note that since all the system state transitions can be captured after each swapping service in the discrete time model, it is convenient to set the length of time slot to be  $D_s$ .** Thus, at time epoch  $k$ , the SS is empty and all batteries of the

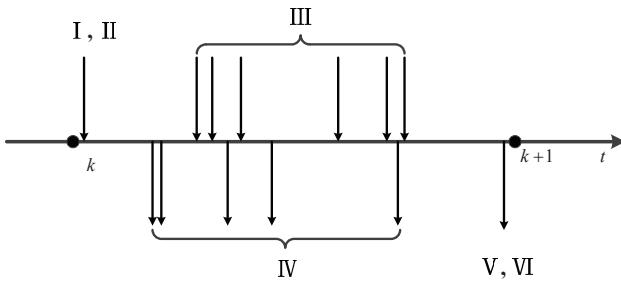


Fig. 2. Timing conventions. In the figure, I and II represent that  $A_2(k)$  EVs enter SS and  $u(k)$  DBs are put into CS just after slot epoch  $k$  respectively.  $A_1(k)$  EV arriving at the BSS and  $A_3(k)$  FCBs leaving CS during time slot are described by III and IV. V stands for  $A_2(k)$  EVs leaving SS and VI shows that  $B_2(k)$  EVs leave waiting queue due to intolerable waiting time just prior to the end of slot.

BSS are in the DB queue, FCB queue or charging servers (CS). According to Fig. 1, the service rate of SS can be described by  $\mu_{ss} = \frac{A_2(k)}{D_s}$ , where  $A_2(k) = \min\{Q_1(k), Q_2(k), N_{13}\}$  denotes the number of batteries arriving at DB queue at the end of time slot  $k$ .

Suppose there are  $N_2$  charging outlets in the CS and system operators are able to decide the number of DBs put into CS at time epoch  $k$ , which is denoted by  $u(k)$ . Speaking of the charging methods, many papers assume that battery charging rate can be a continuous variable. However, it is not practical to control every battery's charging rate continuously below its power rating. In addition, intermittent charging should be avoided as it will shorten the battery lifetime [12]. Therefore, in this paper, different from most of the existing literature, we assume that as long as the DBs are put into the CS, they will be charged at a constant charging rate  $r_0$  until fully charged. Thus, the instantaneous total charging rate is controlled by the number of batteries in the CS. The system performance is comprehensively analyzed in [7] when aggressive actions are taken (i.e., charging as many DBs as possible).

Let us denote  $M_c(k) = B - Q_2(k) - Q_3(k)$  as the number of batteries present in the CS. Thus,  $R_c(k) = M_c(k)r_0$  represents the total instantaneous charging rate. Note that there will be  $A_3(k)$  batteries finishing charging during time slot  $(k, k+1]$ . Thus, the dynamics of the waiting queue, DB queue and FCB queue are as follows,

$$Q_1(k+1) = Q_1(k) - A_2(k) + A_r(k) - B_2(k), \quad (1)$$

$$Q_2(k+1) = Q_2(k) - u(k) + A_2(k), \quad (2)$$

$$Q_3(k+1) = Q_3(k) - A_2(k) + A_3(k), \quad (3)$$

where  $A_r(k) = \min\{A_1(k), M - Q_1(k) + A_2(k)\}$  is the actual number of EVs entering BSS during  $(k, k+1]$ . And  $A_1(k)$  is the number of EV arrivals that are willing to get into the BSS but may be blocked due to the limited parking lots capacity.  $B_2(k)$  denotes the departure number of impatient customers from the waiting queue, which will be specified in II-B. The

range of queues are confined by

$$\begin{aligned} 0 &\leq Q_1(k) \leq M, 0 \leq Q_2(k) \leq B, 0 \leq Q_3(k) \leq B, \\ B - N_2 &\leq Q_2(k) + Q_3(k) \leq B. \end{aligned} \quad (4)$$

Intuitively, action  $u(k)$  is constrained by the number of DBs and the spare charging outlets in CS,

$$0 \leq u(k) \leq \min\{Q_2(k), N_2 - B + Q_2(k) + Q_3(k)\}. \quad (5)$$

The following two parts will address two important measurement metrics for the BSS model, based on which our objective is to achieve the trade off between charging cost and QoS.

### B. Service Loss Probability

One critical issue BSSs have to consider is the QoS. The BSS must guarantee most of the EVs coming for battery swapping service can be served and the mean waiting time is relatively small. Therefore, the QoS can be measured by service loss rate which comes from the following two aspects.

- Type One.** New EV arrival comes and finds that the  $Q_1(k)$  is full. Let  $B_1(k) = A_1(k) - A_r(k)$  be the type one service loss number within  $(k, k+1]$ .
- Type Two.** The drivers in the parking lots cannot tolerate the waiting time for battery swapping service and choose to leave the system. The type-two loss number is denoted by  $B_2(k)$ . We assume that the customers will leave if they are kept waiting for more than  $L_w D_s$  seconds. Let  $R(k)$  denote the number of remaining EVs that arrived during time interval  $(k - L_w, k - L_w + 1]$  before the type-two loss within  $(k, k+1]$  happens.  $R(k) = Q_1(k - L_w + 1) - \sum_{l=L_w-1}^0 A_2(k-l) - \sum_{l=L_w-1}^1 B_2(k-l)$ . Thus, we have  $B_2(k) = \max\{0, R(k)\}$ .

Thus, the service loss probability can be formulated as

$$P_b = \frac{1}{\mathbb{E}[A_1(k)]} \lim_{K \rightarrow \infty} \frac{1}{K} \mathbb{E} \left\{ \sum_{k=0}^{K-1} [B_1(k) + B_2(k)] \right\},$$

where  $\mathbb{E}[A_1(k)]$  denotes the time average EV arrivals (e.g., for stationary Poisson arrivals, this term equals to the arrival rate) and the latter part of  $P_b$  denotes the time average blocked EVs. In order to guarantee the QoS of the BSS,  $P_b$  has to be restricted by

$$P_b \leq \varepsilon_b, \quad (6)$$

where  $\varepsilon_b$  is the predetermined tolerance of the system.

### C. Cost Function

Instantaneous peak charging rate is another big issue that BSSs must pay attention to. It is known that a high instantaneous charging rate brings great burden to the electricity generation and may exceed the operating power limit of cables and transformers during electricity distribution. Therefore, similar to the conventional assumption in the literature (e.g. see [12]), we define the charging cost as a general quadratic function  $C_c(k) = \alpha(R_c(k))^2 + \beta R_c(k)$  within each time slot. Thus,

the optimal battery charging operation can be formulated to minimize the long-term average cost, which is given by

$$\begin{aligned} \min_{u(k)} \quad & \lim_{K \rightarrow \infty} \frac{1}{K} \mathbb{E} \left\{ \sum_{k=0}^{K-1} (C_c(R_c(k))) \right\}, \\ \text{s.t.} \quad & (1) - (6). \end{aligned} \quad (7)$$

It is easy to see that Problem (7) is a constrained MDP (CMDP) problem, which is hard to solve in general. Moreover, due to the highly coupling of the open queue and the closed queue as well as the randomness of the EV arrivals, battery SoC and charging time, directly solving Problem (7) to its optimality is extremely difficult if not impossible. Nevertheless, in the next section, we will demonstrate an effective approach to transform the original CMDP problem into a conventional MDP problem, by which the problem can be tackled by dynamic programming. We will also briefly show that the problem considered in this paper can be easily extended to a multi-time scale MDP problem.

### III. STOCHASTIC CONTROL FORMULATION

Recall that the control action of Problem (7) for the BSS operators is to decide the number of DBs put into the CS at the beginning of each time slot. The randomness mainly comes from the departure process of charging servers and arrival process of EVs. In this paper, we assume that the charging time for each DB is exponentially distributed. EV arrival process is influenced by time instances, BSS location, weather and many other factors. We model the EV arrival process as Poisson arrival process and consider two types of settings in this section. III-A focuses on the battery charging operation when the EV arrival process is stationary with average arrival rate  $\lambda$ . In the sequel, we extend our model to consider non-stationary EV arrival in III-B. Note that both the arrival and charging distribution assumptions are mild and they are also widely used in the queueing theory related literature.

#### A. Charging Strategy under Stationary Arrival Rate

For the individual BSS queueing network system, the state is defined as a triple  $S(k) = \{Q_1(k), Q_2(k), Q_3(k)\}$ . Let  $s = \{q_1, q_2, q_3\}$  denote the present state and  $s' = \{q'_1, q'_2, q'_3\}$  denote the next state, further denote  $\mathcal{S}$  as the state space, i.e.,  $s, s' \in \mathcal{S}$ . Note that the state space is restricted by (4). From (5), we know that the action space  $\mathcal{U} = \{u \mid 0 \leq u \leq \min\{N_2 + q_2 + q_3 - B, q_2\}\}$ . Based on (1) - (3), state transition functions can be described as follows,

$$\begin{aligned} q'_1 &= q_1 + a_r - a_2, \\ q'_2 &= q_2 - u + a_2, \\ q'_3 &= q_3 + a_3 - a_2. \end{aligned}$$

The probability that there are  $a$  EV arrivals during one time slot is given by

$$P_A(a) = \frac{(\lambda D_s)^a}{a!} e^{-\lambda D_s}, a \geq 0.$$

The probability that  $d$  batteries finish charging within one time slot can be represented by

$$P_D(d) = \binom{M_c}{d} (1 - e^{-\frac{D_s}{D_c}})^d e^{-\frac{D_s}{D_c}(M_c + u - d)},$$

in which we assume at most one battery can be fully charged within one time interval. Then, the transition probability

$$\begin{aligned} P(s'|s, u) &= P(q'_1, q'_2, q'_3 \mid s, u) \\ &= P(q'_2 \mid s, u) P(q'_1 \mid q'_2, s, u) P(q'_3 \mid q'_1, q'_2, s, u), \end{aligned}$$

where  $P(q'_2 \mid s, u) = 1$ ,  $P(q'_3 \mid q'_1, q'_2, s, u) = P_D(q'_3 - q_3 + a_2)$ , and  $P(q'_1 \mid q'_2, s, u)$  is given by

$$P(q'_1 \mid q'_2, s, u) = \begin{cases} P_A(q'_1 - q_1 + a_2) & 0 \leq q'_1 < M \\ \sum_{k=M-q_1+a_2}^{+\infty} P_A(k) & q'_1 = M \end{cases}$$

Therefore, the transition probability is

$$\begin{aligned} P(s'|s, u) &= \mathbb{I}_{\{q_3 - a_2 \leq q'_3 \leq B - q_2 + u - a_2, q'_2 = q_2 - u + a_2\}} \\ &\quad P_D(q'_3 - q_3 + a_2) \left\{ \mathbb{I}_{\{q_1 - a_2 \leq q'_1 < M\}} P_A(q'_1 - q_1 + a_2) \right. \\ &\quad \left. + \mathbb{I}_{\{q'_1 = M\}} \sum_{k=M-q_1+a_2}^{+\infty} P_A(k) \right\}, \end{aligned} \quad (8)$$

where  $\mathbb{I}_{\{A\}}$  is an indication function which equals to 1 if  $A$  is true and 0 otherwise.

Constraint (6) captures the fact that in order to prevent new comers from blocking and avoid impatient leaving, BSS operators prefer to charge batteries conservatively when the EV arrival traffic is light and charge aggressively as the waiting queue comes near its maximal capacity. And this control strategy can be achieved approximately by adding a waiting cost function  $C_w(Q_1(k))$  to the objective function. Then, the one stage total cost during time slot  $(k, k+1]$  can be denoted by  $C(s, u, k) = C_c(R_c(k)) + \eta C_w(Q_1(k))$ . Note that  $C_w(Q_1(k))$  is parameterized by the waiting queue length which is assumed to be monotonically increasing and strictly convex in the queue length (e.g. an exponential function), and  $\eta$  is a weighting factor balancing the charging cost and waiting cost. Therefore, the objective function is reformulated as

$$J^* = \lim_{K \rightarrow \infty} \frac{1}{K} \mathbb{E} \left\{ \sum_{k=0}^{K-1} C(s, u, k) \right\}. \quad (9)$$

Note that by this transformation, it is equivalent to say that the QoS constraint (6) is relaxed to the objective function with a specific Lagrangian multiplier. It is known that when constraint (6) is binding, the aforementioned transformation is lossless; otherwise, it is always easy to tune the weighting factor  $\eta$  to achieve a targeted QoS by, for example, the bisection method.

Searching for an optimal charging operation policy falls into the field of MDP problems. The decision policy is a mapping  $\pi : \mathcal{S} \rightarrow \mathcal{U}$ , which guides the BSS operators to make decision sequentially once observing the system state. Let  $\Pi$  denote the set of all possible policies. For any initial state  $s \in \mathcal{S}$ , the optimality equation can be described by

$$J^* + V(s) = \min_{\pi \in \Pi} \left\{ C^\pi(s, u) + \sum_{s' \in \mathcal{S}} P^\pi(s'|s, u) V(s') \right\}.$$

Note that in the long-term average case, we eliminate the time index and try to derive a stationary policy to achieve the minimal average cost. The embedded Markov chain is a unichain so that for different initial states, they have the same

average cost.  $V(s)$  is the relative value function that captures the first few stages' average cost difference for different initial states. The optimality equation can be solved by dynamic programming with convergence guarantee according to [14].

### B. Charging Strategy under Non-Stationary Arrival Rate

In the last subsection, we assume that dynamics of EV arrival is steady with a constant average arrival rate. However, in practice, average EV arrival rate typically varies with time within one day. Therefore, in this subsection, we extend our model to further take the non-stationarity of EV arrivals into consideration. To model the practical arrival process, we assume that each hour has different average arrival rate but within each hour, the arrival rate remains unchanged. *The fluctuation of the EV arrivals among different hours within one day and the randomness of the EV arrivals within the same hours of different days make it impossible to find an optimal stationary policy.* To better represent the nature of the problem, we switch to find the optimal non-stationary charging policy for each day by using the finite horizon MDP model. Therefore, the operator makes new schedule every day based on the estimation of next day's EV traffic profile. Note that the optimal policy will vary with time, which makes the practical implementation of the finite MDP policy more complex.

Different from the infinite horizon MDP model in the last subsection, we include the time space as an additional state space, which is denoted by  $\mathcal{H} = \{0, 1, 2, \dots, 23\}$ . For any  $h \in \mathcal{H}$ , it represents the time index for one hour. Further let  $\lambda_h$  be the average EV arrival rate in hour  $h$ . It should be made clear that within a particular hour  $h$ , the EV arrival process is captured by the Poisson arrival rate  $\lambda_h$ . Therefore, the transition probability for each certain hour  $P(s'|s, u, h)$  can be obtained by slight modification of (8). Similarly, the optimality equation can be described by

$$V(s, h, k) = \min_{\pi \in \Pi} \left\{ C^\pi(s, u, h, k) + \sum_{s' \in \mathcal{S}} P^\pi(s'|s, u, h) V(s', k+1) \right\},$$

where  $V(s, h, k)$  is the cost-to-go function from the dynamic programming perspective. The optimal policy to minimize the total cost of the finite horizon can be easily obtained by backward induction based on the above optimality equation.

**Remark 1.** *In our finite MDP model, the state evolves in an hourly time-scale, while the battery charging operation decision should be made in time scale of minutes. Actually, when the different states or actions processes evolve in different time scales, the MDP problem falls into the field of multi-time scale MDP (MMDP) [13]. For regular MMDP, the slow time scale decisions will affect the fast time scale decisions and the fast time scale decisions in turn will affect the decisions made by slow time scale. However, in our case, the slow time scale state evolves deterministically into next hour, which is free from the effect of fast time scale decisions. Therefore, any local optimal policy within each hour is also global optimal.*

## IV. CASE STUDY AND DISCUSSION

In this section, we evaluate the performance of infinite time horizon policy and finite time horizon policy for different scenarios. Charging cost and blocking rate are used as performance metrics to evaluate the BSS operation cost and QoS, respectively. In our simulation, we assume that the BSS has already been built and system parameters are set as follow. BSS has  $M = 5$  parking places,  $N_{13} = 3$  swapping islands and  $N_2 = 8$  charging outlets. The battery swapping time  $D_s = 1$  minute and the average service time for each charging server  $D_c = 20$  minutes. In different scenarios, the total battery number  $B$  is set within the range  $[10, 15 \dots 40]$  and the average arrival rate  $\lambda$  varies from 0.1 EV/min to 0.5 EV/min.

### A. Infinite Time Horizon Average Cost Policy Operation

In this part, we first investigate the influence of battery number on the average charging cost and then discuss the charging outlets number selections under different traffic environment.

In Fig. 3, we show the blocking rate and average cost with increasing total battery number in different traffic load environment. Note that the blocking rate and average cost both decrease in the total battery number while increase in the EV arrival rate. Owning more batteries can give the BSS system operators more flexibility to pre-charge enough batteries at a relatively lower instantaneous charging rate and ensure it can offer enough FCBs under heavy traffic. Moreover, from the long-term average perspective, the average effective EV arrival rate is equal to the actual charging rate.  $(1 - P_b)\lambda = \sum_{i=1}^{|S|} \frac{(B - q_2(i) - q_3(i) + u^*(s(i)))}{D_c} P_{\pi^*}(i)$  where  $u^*(s(i))$  is the optimal action for state  $s(i)$  and  $P_{\pi^*}(i)$  represents the steady state probability distribution for the optimal policy controlled embedded Markov chain. If the actual EV arrival rate exceeds the maximal service rate  $\frac{N_2}{D_c}$ , BSS can never satisfy all the EV arrivals. Thus, the waiting queue will always be nearly full and all the charging outlets will always be busy, which means the system operators have no freedom to take actions. In our simulation, when  $\frac{N_2}{D_c} = 0.4$  battery/min, the blocking rate increases rapidly when the arrival rate approaches or exceeds 0.4 EV/min. The system operators have to construct enough charging outlets to avoid this type of scenarios in practice.

### B. Finite Time Horizon One-day Policy Operation

Now we start to consider the practical EV arrival profile and compare the performance of the following three policies.

- **Stationary Policy (SP).** We adopt the long-term average cost policy. Its constant average arrival rate is equal to the average of the 24-hour variable arrival rate within one day.
- **Non-Stationary Policy 1 (NSP1).** This is the optimal policy obtained from our finite horizon MDP model by the standard backward induction with zero-valued initial value function.
- **Non-Stationary Policy 2 (NSP2).** In order to eliminate the unknown effects of the initialization of the backward induction, we assume each day of the following one week follows the same EV arrival profile and extends the schedule horizon to one week. We take the first day's policy as the optimal operation for that particular day.

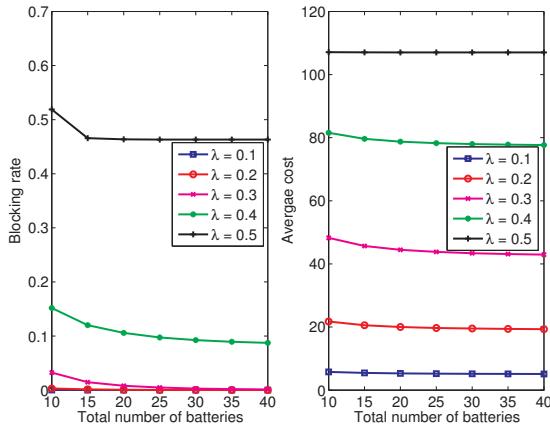


Fig. 3. Blocking rate and average cost for BSS. The figure describes the effects of total battery number and average EV arrival rate for the long-term average charging policy.

Monte Carlo simulation is applied to produce the realizations of operating process under different policy controls. The blocking rate and cumulative charging cost for different scenarios are shown in Table I and Fig. 4, respectively. We can find the blocking rate of SP and NSP1 decrease with  $N_2$ . Furthermore, the NSP1 decreases more quickly than SP. Note that when  $N_2$  is large enough, the one-day cumulative charging cost of NSP1 is lower than that of SP and meanwhile, its blocking rate is smaller than SP. In fact, NSP1 outperforms SP because it knows more detailed EV arrival variations for different hours, which means it takes into account more information in the dynamic environment.

According to the results above, if we want to reduce charging cost and guarantee a low blocking rate, we need to acquire more accurate EV arrival information. Furthermore, by leveraging the modern communication technology, BSSs are able to deliver their system state information (e.g. available FCBs) to guide EV arrival behaviors. Thus the whole BSS network can work together and make cooperative charging schedule to help the grid work more efficiently.

TABLE I  
BLOCKING RATE FOR DIFFERENT SCENARIOS

	SP	NSP1	NSP2
$N_2 = 6, N_2/D_c = 0.3$	0.43191	0.56749	0.56664
$N_2 = 8, N_2/D_c = 0.4$	0.13184	0.27546	0.27440
$N_2 = 10, N_2/D_c = 0.5$	0.03982	0.02741	0.02700

## V. CONCLUSIONS

In this paper, we proposed a novel queueing network model for individual BSS operation. The EVs were modeled as an open queue that was shown to couple with the closed queue of batteries by sharing the same SS. We have obtained the optimal charging policy with QoS guarantee by dynamic programming. In order to keep a low blocking rate, sufficient number of total batteries and charging outlets are required. Monte Carlo simulations also demonstrated that better decisions could be obtained by more accurate non-stationary modeling methods.

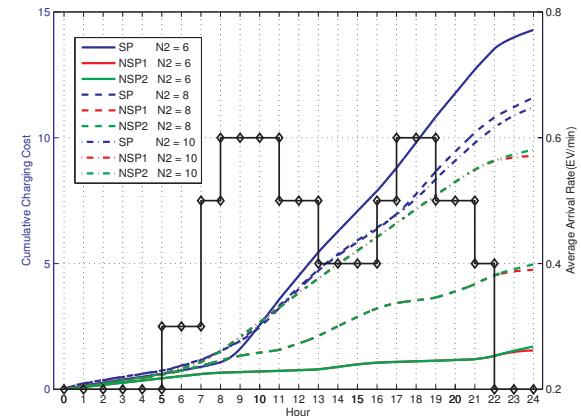


Fig. 4. Cumulative charging cost for different scenarios. The black curve shows the variable EV arrival rates in different hours and the others describe the cumulative charging cost of different scenarios in one day.

Our work can be a benchmark for future BSS network design and operation.

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