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Optimal Scheduling for Electric Vehicle Charging with Discrete Charging Levels in Distribution Grid

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***Abstract*—To accommodate the increasing electric vehicle (EV) penetration in distribution grid, coordinated EV charging has been extensively studied in the literature. However, most of the existing works optimistically consider the EV charging rate as a continuous variable and implicitly ignore the capacity limitation in distribution transformers, which both have great impact on the efficiency and stability of practical grid operation. Towards a more realistic setting, this paper formulates the EV coordinated discrete charging problem as two successive binary programs. The first one is designed to achieve a desired aggregate load profile (e.g., valley-filling profile) at the distribution grid level while taking into account the capacity constraints of distribution transformers. Leveraging the properties of separable convex function and total unimodularity, the problem is transformed into an equivalent linear program, which can be solved efficiently and optimally. The second problem aims to minimize the total number of on-off switchings of all the EVs' charging profiles while preserving the optimality of the former problem. We prove the second problem is NP-hard and propose a heuristic algorithm to approximately achieve our target in an iterative manner. Case studies confirm the validity of our proposed scheduling methods and indicate our algorithm's potential for real-time implementations.**

***Index Terms*—EV charging, discrete charging level, load valley-filling, binary quadratic program, total unimodularity.**

I. Introduction

As the environmental pollution and fossil fuel scarcity incur increasing concern all over the world, electrification of transportation has attracted a wide range of attentions from government, industry and academy. Electric vehicles (EVs) emerge as promising components to substitute the conventional vehicles in the future smart grid [1][2]. Correspondingly, how to accommodate the large-scale EV penetration with stable and convenient energy support becomes a crucial issue for both the power grid operators and the government policy makers.

Currently, the main EV refueling techniques can be categorized into two approaches. The first approach is the slow but economic EV charging that happens in places such as homes, parking lots and street charging spots, where EVs can be left idle for a relatively long time without emergent refueling requirement. In contrast, the other refueling method is provided in some specific EV refueling stations that are capable of fulfilling fast driving range extension within a time duration compatible to refueling a traditional gasoline vehicle. Though some leading companies (e.g., Tesla) have proposed super charging and battery swapping techniques which can shorten the refueling time significantly [3], the wide deployment of infrastructure with such techniques not only requires time to be completed but still takes time to reach adequate market demand. Thus, slow EV charging overnight in residential areas is still expected to be the first choice of most individual EV owners in the near future [4][5].

However, large-scale slow charging in residential areas introduces significant electricity consumption and may bring harmful large peaks to existing distribution gird [2][4]. Although the increase of total energy demand can be supported by gradually upgrading infrastructure capacity, unexpected large load peaks would require adequate backup of expensive fast generators, increase power losses of transmission/distribution lines, and frequently overload grid components (e.g., transformers and cables) especially in weak distribution grid [2][4][7]. Therefore, from the grid operator's point of view, the EV charging is expected to be coordinated so that the total energy consumption, including the base load and EV charging load, can be shaped to achieve a desired total load profile at the distribution level, which helps maintain the energy efficiency and grid stability. On the other hand, participating in charging coordination is also beneficial for EV owners. Typically, EV owners are flexible with charging time but expect to lower down their electricity bills as long as their EVs can be charged to their target SoC levels before certain deadlines. Thus, the coordination of EV charging offers EV owners the opportunity to bid in the electricity market as a whole for achieving lower charging cost.

Motivated by the above reasons, the coordination of EV charging has been extensively studied in recent years (to be reviewed in the next section). In particular, most of the existing works consider the coordination of EV charging based on the assumption that EVs can adjust their charging power continuously between zero and their maximum charging rates (i.e., continuous charging). However, due to the limitations of the current battery technology (e.g., the lithium-ion battery) and EV charger technology (e.g., the constant-current constant-voltage approach [6]), EVs can only draw an approximately constant power during charging periods (i.e., discrete charging) [5][6][16]. Although the continuous charging method is promising to be commercialized in the future, we envision that the discrete charging method will still be the dominating one in the near future, and will co-exist with the continuous

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charging method in the long run[1]. Therefore, it is of practical importance to investigate the coordination of EV charging based on the discrete charging method, and this motivates our work.

In summary, our contributions in this paper are three-folds. i) We propose an offline EV coordinated discrete charging model with grid capacity constraints. In particular, the model is formulated as two successive binary programs that aim to optimize the total load variations and total number of on-off switchings in the charging process, respectively. Our optimal solution not only serves as a benchmark for the EV discrete charging with grid capacity constraints but also provides insights to derive efficient algorithms for real-time implementations; ii) The EV coordinated discrete charging problem is transformed into an equivalent linear program (LP) leveraging the underlying properties of its separable convex objective and totally unimodular constraint coefficient matrix. Thus, the first binary program can be solved efficiently and optimally by just solving an LP; iii) The second binary program is proven to be NP-hard in the strong sense. Hence, we design a heuristic algorithm to merge charging periods of each EV to prevent undesirable frequent on-off switching during the EV charging process.

II. Related Work

We consider the EV coordinated charging problem from an optimization perspective. Therefore, we first review the related literature on how to formulate the EV coordinated charging problem, and then further survey the directions of algorithmic development for EV coordinated charging.

Typically, there are three dimensions to control the charging process in EV coordinated charging problems: space (which EV to charge), time (when to charge) and speed (at what rate to charge). Most of the existing works [8]-[15] choose the continuous charging rates as their decision variables, which can affect the charging time and the charging speed for each EV. Such problems have been formulated as linear [8]-[10] or convex quadratic [11]-[15] programs for various objectives and can be solved by either centralized [8]-[12] or decentralized [13]-[15] methods leveraging extensive convex optimization techniques. However, due to the limitation of the charging circuit, the continuous charging rate is difficult to be implemented and chargers in current practice can only support several discrete charging levels as mentioned in the previous section. There are a rather limited number of papers discussing the potential problems that may be induced when the EV charging rate is discrete. The works [16][17] consider the uninterruptible discrete charging case, and the proposed optimization problems try to decide the optimal instance to start charging for each EV. In particular, a decentralized randomized algorithm is designed in [16], which solves the problem in an iterative manner and its suboptimal ratio is theoretically derived and proven. However, such algorithms suffer from heavy computation and communication overheads. [17] proposes a scalable greedy algorithm to lower down the computational complexity. However, the optimality of the algorithm cannot be guaranteed. In addition to the aforementioned literature, the space dimension in the decision space is constantly ignored. For example, the papers [12]-[14], [16][17] implicitly assume that charging performances are independent of EV locations. However, it is usually not the case in practice because specific charging locations can greatly affect the congestion conditions over the weak distribution transformers. To avoid exceeding the transformer capacities, [9] iteratively maximizes the network flow and finds a feasible solution in a centralized manner, but it is computationally expensive and lacks optimality guarantee. [10][11] explore the impacts of EV charging on the distribution transformers in more details and schedule the EV charging process centrally taking the capacity constraints into consideration. To reduce the computational complexity, [15] uses the ADMM technique to include the network capacity constraints in its decentralized algorithm, but the proposed method induces more communication overheads to achieve an optimal solution.

The recent follow-up papers [18]-[21] on the EV coordinated charging problem mainly focus on two important directions: how to make the control algorithms scalable for the increasing population of EVs, and how to design real-time/online algorithms to mitigate the impacts of uncertainties from the EVs (e.g., plug-in time, energy demand) in practical implementations. Most of the existing works [13]-[15], [18]-[20] achieve scalability by designing decentralized algorithms based on their corresponding centralized algorithms. While other works try to design new control architectures to derive scalable algorithms. For example, [21] decomposes the centralized problem into three steps, and optimization is only performed in one of the steps to obtain the optimal aggregate load profile. Subsequently, the optimal aggregate load is distributed efficiently in a market-based mechanism among all the EVs. In order to design online algorithms for coordinated EV charging, the main problem is how to model and integrate uncertain EV load into the real-time decision process. To address this issue, [22] proposes to scale up the total EV load by a properly chosen factor when doing the scheduling to compensate for the underestimation of the future EV load. Instead of simply scaling up, other works estimate the future load by simulation [20] or analyzing historical data [19][21]. Specially, it can be proven that the suboptimality of the algorithm in [20] vanishes as the time horizon increases.

In the literature, it is a common way to design online or decentralized algorithms based on their corresponding offline centralized formulations. Convexity of the centralized problem plays an important role in guaranteeing the low computational complexity for the online algorithms and optimality for the decentralized algorithms. However, when considering the physical constraints (e.g., the power flow constraints [19], the discrete charging rate [16]) in the EV coordinated charging problem, the centralized problem becomes non-convex and the corresponding online decentralized algorithms with perfor-

[1]This is because both the discrete charging method and the continuous charging method have their own advantages and disadvantages. For example, the discrete charging method is easier to be implemented because it only requires a simple on/off controller with communication capability. However, it is less flexible for providing grid services. In comparison, the continuous charging method is more flexible but requires more sophisticated and expensive control devices.

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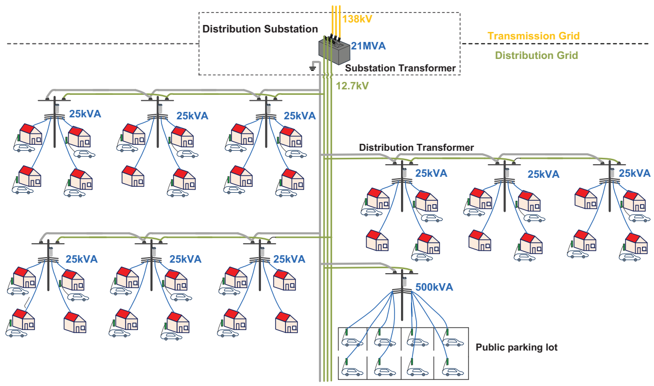

Fig. 1. Illustration of distribution grid in North America.

mance guarantee will be difficult to achieve. To cope with the non-convexity, convex relaxation [19] and randomization [16] are introduced to derive distributed algorithms. Our problem in the rest of the paper is an offline centralized non-convex problem, and our major focus is to show how to transform this non-convex problem into an equivalent LP, which is the simplest convex function. Thus, online decentralized algorithms can be designed by leveraging the convexity of the equivalent LP.

III. System Model and Problem Formulation

Consider the scenario that a trustworthy EV coordinator takes over the charging operation of all the EVs in the downstream of a distribution substation. The control horizon is divided into slots of equal duration Δt (e.g., 15 minutes). Let $\mathcal{T} = \{1, 2, \ldots, T\}$ denote the set of time slot indexes. Fig. 1 illustrates a typical distribution network in North America [23], in which high-voltage electricity from transmission grid will be stepped down twice by the respective substation transformer and distribution transformers before it eventually reaches houses or public electrical facilities (e.g., public parking lots) in the residential area. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\mathcal{M} = \{1, 2, \ldots, M\}$ denote the set of EVs and load buses (i.e., distribution transformers). According to the topology of the distribution grid, $\mathcal{N}$ is divided into M disjoint subsets $\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_M$, where $n \in \mathcal{N}_m$ if EV n is connected to load bus m.

A. Assumptions

1) Offline information: We assume that the EV coordinator is able to obtain the following information at the beginning of the decision horizon.

- Topology of the distribution grid: $\mathcal{N}_m$ and $\mathcal{M}$.
- Estimation of the base load on bus m at time t: $D_m(t)$.
- EV charging specifications: plug-in time t_n^i, plug-off time t_n^d, energy demand e_n. The energy demand e_n represents the number of time slots to charge EV n to its desired SoC level.

Acting as a cooperator of the grid operator, the EV coordinator can typically have access to the grid-side information (e.g., topology and base load estimation) in order to maintain the grid stability during the EV charging process. In addition, for private EVs that are considered in this paper, their charging specifications can be predicted from the history charging profiles with a reasonable accuracy according to the characteristics of specific drivers' driving habits and lifestyles. For example, on a working day, an EV owner stops charging and leaves for work at 8 a.m. and plug in their EVs after coming back home at 6 p.m. with the energy consumed by commuting on that day as his/her energy demand.

2) Single charging rate: Due to the trend to standardize the charging equipment and limitation of charging technology [5][6][16], EVs are first assumed to be charged at a constant charging rate r_0. The extension for the problem of multiple charging rates will be discussed in Section IV.

B. Problem Formulation

An EV is said to be connected to the grid during the periods between its plug-in and plug-off times. Let $I_{n,t}$ be an indicator function which equals 1 if EV n is connected at time t and 0 otherwise. For the single charging rate case, the basic problem of EV coordinated charging is to decide whether to charge or not for each EV during the periods it is connected. Specially, the decision variables are denoted by the scheduling matrix $\mathbf{U}$, where each entry of $\mathbf{U}$ is denoted as follows:

$$u_{n,t} = \begin{cases} 1 & \text{if } I_{n,t} = 1 \text{ and EV } n \text{ is charging at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Note that EVs can only be charged when they are connected.

The objective of the EV coordinator is to reshape the total load of the whole distribution grid to track some predetermined load profile $L(t)$, $\forall t \in \mathcal{T}$ (e.g., [13][16]). Typically, $L(t)$ is designed to either maximize the economic benefit in the electricity market or minimize the operating cost under the requirement of grid stability so that a triple-win result for the EV coordinator, the grid operator and the EV owners can be realized. Following the literature [13], the objective function can be formulated as

$$F(\mathbf{U}) = \sum_{t=1}^{T} f_t\Big(r_0 \sum_{n=1}^{N} u_{n,t} + D(t) - L(t)\Big),$$

where $D(t) = \sum_{m=1}^{M} D_m(t)$ denotes the total base load of the distribution grid. $f_t(\cdot)$ is a time-dependent convex function measuring the cost of deviating from target load profile $L(t)$ and $f_t(0) = \min_x f_t(x) = 0$. We illustrate two classes of target load profiles $L(t)$ and their corresponding functions $f_t(\cdot)$ as follows.

- The EV coordinator acts as the utility company and participates in the electricity market to minimize its electricity cost by controlling EVs' energy consumption profile. For example, the electricity market in [26] is divided into two stages, where the EV coordinator first determines $L(t)$ as the total energy consumption of the distribution grid in the day-ahead market and buys deficient (or sells superfluous) electricity in the balance market at additional cost. In Fig. 2, both piecewise-linear and deadzone-linear functions can model the penalty of electricity imbalance in the balance market. Deadzone exists if the EV coordinator owns some energy storage devices to compensate its load fluctuation.

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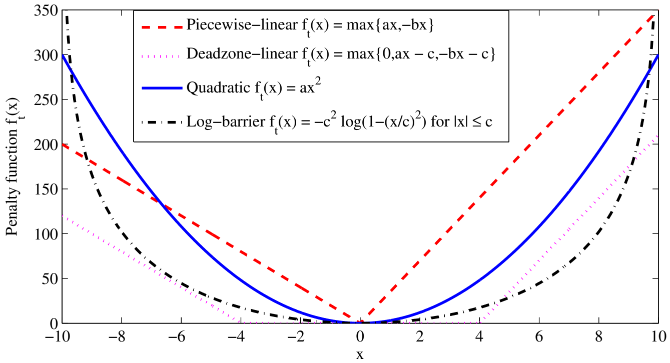

Fig. 2. Illustration of penalty functions. x is the difference between the resulting total load profile and target load profile.

- The EV coordinator is regulated by the grid operator and tries to minimize the variations of the total load profile. In this case, the load profile $L(t)$ is designed to be the average load over the whole time horizon (i.e., $L(t) = \frac{1}{T}\sum_{t=1}^{T}(r_0\sum_{n=1}^{N}u_{n,t}+D(t))$) and the resulting total load profile is well-known to possess the valley-filling property [13][16]. Quadratic and log-barrier functions in Fig. 2 can be used to measure the cost of deviation from the average load profile. Quadratic functions are a typical mean square measurement and log-barrier functions penalize small fluctuations moderately but restrains the deviations strictly within a certain range.

From the EV owners' point of view, one fundamental requirement for the EV coordinated charging is to fulfill each EV's energy demand before it plugs off. Mathematically, this constraint can be captured by

$$\sum_{t=t_n^i}^{t_n^d} u_{n,t} = e_n, \quad \forall n \in \mathcal{N}. \tag{1}$$

Equation (1) guarantee that each EV is allocated enough time slots to charge to its desired SoC level during its connected time periods.

From the grid operator's perspective, stability is the key issue to be considered. In the distribution grid, distribution transformers are usually regarded as the most vulnerable components. In particular, a distribution transformer in North America typically serves approximately 10 houses and only has a limited power capacity of 25 kVA [23]. Such transformers will be easily overloaded when multiple EVs are connected to the same transformer and charged at the same time. Hence, in order to avoid overloading the distribution transformers, the following capacity constraints have to be respected

$$r_0 \sum_{n\in\mathcal{N}_m} u_{n,t} + D_m(t) \le C_m, \quad \forall m \in \mathcal{M}, \forall t \in \mathcal{T},$$

where C_m denotes the capacity of the distribution transformer m. After simple manipulations, the above constraints can be transformed to

$$\sum_{n\in\mathcal{N}_m} u_{n,t} \le c_{m,t}, \quad \forall m \in \mathcal{M}, \forall t \in \mathcal{T}, \tag{2}$$

where $c_{m,t} = \lfloor \frac{C_m - D_m(t)}{r_0} \rfloor$ and $\lfloor x \rfloor$ is the largest integer not larger than x. Note that constraint (2) correlates the charging process served by the same distribution transformer, which imposes extra difficulties on solving the scheduling problem.

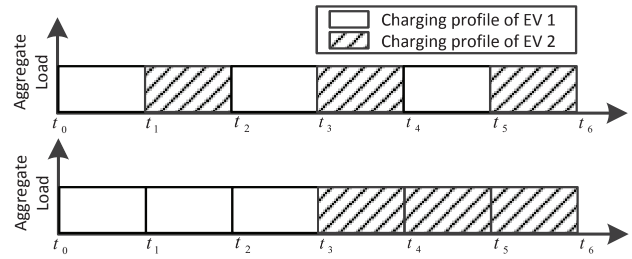

Fig. 3. Illustration of optimal charging profiles with different interruptions. The top and bottom figures depict two optimal charging profiles with the same aggregate load. However, it is clear that the bottom charging profile has less interruptions which is more desired in practice.

In summary, the optimal EV coordinated discrete charging (**OCDC**) problem can be formulated as follows,

$$\begin{aligned} \textbf{OCDC:} \quad & \min_{\mathbf{U}} \quad F(\mathbf{U}), \\ & \text{s.t.} \quad (1), (2), \\ & \qquad u_{n,t} \in \{0,1\}, \quad \forall n \in \mathcal{N}, \forall t \in \mathcal{T}. \end{aligned} \tag{3}$$

Let $\mathcal{U}^*$ denote the optimal solution set of problem **OCDC**. Note that the objective function of the **OCDC** problem is only related to the aggregate load[2] of all the EVs during each time slot. Therefore, normally problem **OCDC** has multiple optimal solutions that produce the same aggregate load. Such problem structure helps define $\mathcal{U}^*$ in a more straightforward manner which will be shown in Section IV.

Besides, another problem for the EV coordinator is whether all the optimal solutions of problem **OCDC** are suitable to be implemented in practice, and if not, how to choose a better one in $\mathcal{U}^*$. In fact, both EV owners and the grid operator prefer to have relatively smooth charging profiles. In other words, the EVs prefer as few on-off switchings as possible during their charging process. The arguments for the smooth charging profiles mainly come from two aspects as follows. i) Frequent interruptions in the charging process may introduce extra deterioration for batteries [16][24]; ii) Switching-on actions will create power spikes on the load buses, which may threaten the stability of distribution transformers [25]. The feasibility of smoothing the charging profile is illustrated in Fig. 3. Intuitively, the smooth **OCDC** (**SOCDC**) problem can be formulated as

$$\begin{aligned} \textbf{SOCDC:} \quad & \min_{\mathbf{U}} \quad G(\mathbf{U}) = \frac{1}{2}\sum_{n=1}^{N}\sum_{t=0}^{T}(u_{n,t}-u_{n,t+1})^2, \\ & \text{s.t.} \quad \mathbf{U} \in \mathcal{U}^*, \end{aligned} \tag{4}$$

where $u_{n,0} = u_{n,T+1} = 0, \forall n \in \mathcal{N}$. The objective function $G(\mathbf{U})$ represents the total number of on-off switchings/interruptions of all the EVs.

Remark 1. In this paper, we focus more on the optimality of problem **OCDC** because an interrupted charging process may

[2]The aggregate load refers to the total load (i.e., the summation of base load and EV charging load) of the distribution network at one specific time slot.

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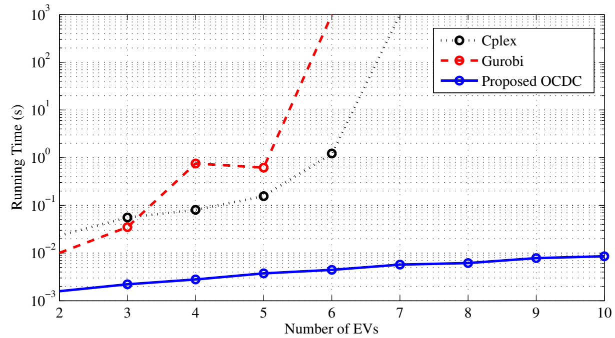

Fig. 4. Illustration of execution time for problem **OCDC**. Time slot duration is set to be 15 minutes and capacity constraints are neglected. Each simulation is repeated 10 times and the average execution time is shown.

not incur any extra cost with the advance of battery/charger technology in the future. Thus, we formulate problem **OCDC** and **SOCDC** individually instead of combining them into a single multi-objective optimization problem. If in the future the battery and the grid can tolerate frequent charging interruptions, then problem **SOCDC** is not needed so we can just solve problem **OCDC** to obtain the optimal coordinated charging solution. It is worth pointing out that the optimal load profile always refers to the optimal solution of problem **OCDC**.

IV. Solution Methodology

In this section, we will show the solution methodology of the EV coordinated discrete charging problems. Problem **OCDC** and **SOCDC** are both binary quadratic programs, which are generally computationally intractable for large-scale (e.g., more than 100 EVs) input instances due to their combinatorial nature. Fig. 4 compares the execution time[3] using our proposed **OCDC** algorithm (to be addressed in detail in Subsection IV-A) and the execution time of commercial solvers (i.e., Cplex and Gurobi) for a simplified version of problem **OCDC**. It is shown that the **OCDC** problem becomes intractable quickly with the increase of EV number. Thus, it is important to design efficient algorithms for our proposed problem leveraging its special structure. In the following part, we first introduce how to transform problem **OCDC** into an equivalent LP. Then, based on the optimal solution of the **OCDC** problem, we show a complete formulation of problem **SOCDC** and prove that problem **SOCDC** is NP-hard in the strong sense. Finally, we propose a heuristic algorithm to search for an optimal charging profile with less interruptions.

A. Optimal Coordinated Discrete Charging

A function is defined as separable convex if it can be represented by a sum of single-variable convex functions. Separability is a desired property for tractable integer programs [28]. To this end, recall that the objective of problem **OCDC** only depends on the aggregate load of each time period. We introduce ancillary variable v_t to denote the total number of EVs in charging mode during time slot t. Then, we have equality constraints as follows.

$$\sum_{n=1}^{N} u_{n,t} = v_t, \quad \forall t \in \mathcal{T}. \tag{5}$$

Let $\mathcal{D}_t$ denote the feasible set of v_t, namely, $\mathcal{D}_t = [0, \overline{v}_t] \cap \mathbb{Z}$, where the upper bound $\overline{v}_t = \sum_{m=1}^{M} \min\{c_{m,t}, \sum_{n\in\mathcal{N}_m} I_{n,t}\}$ depends on the availability of the EVs, the power grid topology and the distribution transformer capacities.

By (5), problem **OCDC** can be reformulated as

$$\begin{aligned} \min_{\mathbf{U},\mathbf{v}} \quad & \sum_{t=1}^{T} f_t(r_0 v_t + D(t) - L(t)), \\ \text{s.t.} \quad & (1), (2), (5), \\ & u_{n,t} \in \{0,1\}, \quad \forall n \in \mathcal{N}, \forall t \in \mathcal{T}, \end{aligned} \tag{6}$$

where $\mathbf{v} = [v_1, v_2, \ldots, v_t, \ldots, v_T]'$, $t \in \mathcal{T}$. $\mathbf{A}'$ represents the transpose of matrix $\mathbf{A}$. Herein, the objective function is the summation of single-variable convex functions $f_t(r_0 v_t + D(t) - L(t))$ and hence separable convex.

Furthermore, total unimodularity is also an important property to eliminate integer constraints without losing optimality. Constraint coefficient matrix with totally unimodular property defines the solution space as a polyhedron, whose vertices are all integral. Thus, total unimodularity helps safely eliminate integer requirement constraints if the optimal solutions are known to be located on the extreme points of the polyhedron (e.g., linear objective).

Theorem 1. *The coefficient matrix of constraints* (1), (2) *and* (5) *is totally unimodular.*

Proof. Please refer to Appendix A. □

Until now, we have shown that problem (6) has a separable convex objective function and totally unimodular constraint coefficient matrix. In general, the convex integer objective function is difficult to tackle. However, leveraging the λ-representation technique [29], a single-variable integer convex function can be replaced by an equivalent LP, which is much easier to handle. Specifically, define a single-variable function $h_t(v_t) = f_t(r_0 v_t + D(t) - L(t)), v_t \in \mathcal{D}_t$, which can be represented as,

$$h_t(v_t) = \min_{\lambda_{t,j}} \sum_{j\in\mathcal{D}_t} h_t(j)\lambda_{t,j}, \tag{7a}$$

$$\text{s.t.} \sum_{j\in\mathcal{D}_t} j\lambda_{t,j} = v_t, \quad v_t \in \mathcal{D}_t \tag{7b}$$

$$\sum_{j\in\mathcal{D}_t} \lambda_{t,j} = 1, \quad \lambda_{t,j} \geq 0. \tag{7c}$$

The λ-representation approximates the continuous function $h_t(\cdot)$ with a piecewise-linear function defined by points $(v_t, h_t(v_t)), v_t \in \mathcal{D}_t$. Due to the convexity of $h_t(\cdot)$, such approximation will be an upper bound of $h_t(\cdot)$ within interval $[0, \overline{v}_t]$ and equals $h_t(\cdot)$ at the integer points in $\mathcal{D}_t$. Given the integrality constraints of problem (6), only the function values

[3]The simulations are implemented on a virtual machine (VM) in our private cloud. The VM gets 20 CPU cores from Intel Xeon ES-2470 v2 processor (2.40 GHz) and 24 GB of memory. Because the execution time of the **OCDC** problem has no theoretical bound, the calculation is forced to stop when the execution time exceeds 1000 seconds.

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at integral points will be counted. Thus, this λ-representation is exactly equivalent to our original single-variable convex function $h_t(\cdot)$ when the decision variable is restricted to be integer. Substituting (7) into (6) and combining the two-level minimization together, problem **OCDC** can be transformed into an integer linear program

$$\begin{aligned} \min_{\mathbf{U},\mathbf{v},\boldsymbol{\lambda}} \quad & \sum_{t=1}^{T}\sum_{j\in\mathcal{D}_t} h_t(j)\lambda_{t,j}, \\ \text{s.t.} \quad & (1),(2),(5),(7b),(7c), \\ & u_{n,t}\in\{0,1\}, \quad \forall n\in\mathcal{N},\forall t\in\mathcal{T}, \end{aligned} \tag{8}$$

where new variables $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \ldots, \boldsymbol{\lambda}_t, \ldots, \boldsymbol{\lambda}_T]'$ and $\boldsymbol{\lambda}_t = [\lambda_{t,0}, \lambda_{t,1}, \ldots, \lambda_{t,j}, \ldots, \lambda_{t,\overline{v}_t}]', j \in \mathcal{D}_t$ are introduced. By utilizing the λ-representation and the properties of totally unimodular constraint coefficient matrix, it is proven in [29] that after relaxing the integrality constraints, the optimal solutions of (8) are guaranteed to be integral and are the same as the optimal solutions of problem **OCDC**. Thus, the **OCDC** problem can be solved efficiently and optimally via algorithms designed to solve LP (e.g., the simplex or interior-points algorithm).

Remark 2. We have so far discussed the single charging rate case. The more general case of multiple charging rates can be a natural extension of our proposed **OCDC** problem, and the same technique presented above can be applied to solve the extended **OCDC** problem if the multiple charging rates are integer multiples of the lowest charging rate r_0. However, for the case that multiple discrete charging rates are arbitrarily selected, the problem becomes far more complicated and thus is left for our future work. Furthermore, our proposed methods to solve the EV charging problem can be naturally extended to coordinate the charging scheduling of residential-level battery energy storage.

B. Smoothing the EV Charging Profiles

Let $(\mathbf{U}^*, \mathbf{v}^*, \boldsymbol{\lambda}^*)$ denote an optimal solution of problem (8). As mentioned before, the optimality of problem **OCDC** is characterized uniquely by $\mathbf{v}^*$, which represents the optimal aggregate number of EVs during each time period. Given $\mathbf{v}^*$, $\mathcal{U}^*$ can be defined by a set of linear equations. Thus, problem **SOCDC** is described by a binary quadratic program as follows.

$$\begin{aligned} \min_{\mathbf{U}} \quad & G(\mathbf{U}) = \frac{1}{2}\sum_{n=1}^{N}\sum_{t=0}^{T}(u_{n,t}-u_{n,t+1})^2, \\ \text{s.t.} \quad & (1),(2), \\ & \sum_{n=1}^{N} u_{n,t} = v_t^*, \quad \forall t\in\mathcal{T}, \\ & u_{n,t}\in\{0,1\}, \quad \forall n\in\mathcal{N},\forall t\in\mathcal{T}. \end{aligned} \tag{9}$$

Note that the coefficient matrix of problem (9) is a submatrix of problem **OCDC**'s coefficient matrix and hence is totally unimodular. However, the objective function of problem (9) is not separable, which makes it inappropriate to solve in the same way as we did in the last subsection. In fact, problem (9) is proven to be NP-hard in the strong sense.

TABLE I
ILLUSTRATING THE VALIDITY OF THE LINEARIZED OBJECTIVE

t	1	2	3	4	5	6	7	8	9
$u_{n,t}^{(k)}$	1	0	1	1	1	0	0	1	0
$\alpha_{n,t}^{(k)}$	-6	-11	-16	-17	-7	-1	-10	-6	-1
$u_{n,t}^{(k+1)}$	0	1	1	1	1	0	1	0	0
$\alpha_{n,t}^{(k+1)}$	-10	-16	-17	-17	-7	-11	-6	-1	0
$u_{n,t}^{(k+2)}$	1	1	1	1	0	1	0	0	0
$\alpha_{n,t}^{(k+2)}$	-16	-17	-17	-7	-11	-6	-1	0	0
$u_{n,t}^{(k+3)}$	1	1	1	1	1	0	0	0	0

Theorem 2. *Problem (9) is NP-hard in the strong sense. There can be no polynomial time approximation algorithms for problem (9) unless P=NP.*

Proof. Please refer to Appendix B. □

SOCDC Algorithm

Input: Distribution grid topology $\mathcal{N}_m$, base load D_m and capacity C_m for each bus $m \in \mathcal{M}$. EV specification (t_n^i, t_n^d, E_n) for each EV $n \in \mathcal{N}$.
Output: Charging profile $\mathbf{U}^b$.
Solve problem (8) and obtain optimal solution $(\mathbf{U}^*, \mathbf{v}^*)$.
Set $k = 0$, $k^{(\max)} = 100$. Initialize the charging profile by having $\mathbf{U}^b = \mathbf{U}^{(0)} = \mathbf{U}^*$.
repeat
$\mathbf{U}^{(k+1)} = \arg\min_{\mathbf{U}\in\mathcal{U}^*} G^l(\mathbf{U}, \mathbf{U}^{(k)})$.
$\mathbf{U}^b = \arg\min_{\mathbf{U}\in\{\mathbf{U}^b,\mathbf{U}^{(k+1)}\}} G(\mathbf{U})$.
$k = k + 1$.
until convergence or $k = k^{(\max)}$.

Thus, we turn to finding a simple heuristic algorithm which can achieve reasonably good results within a proper time. Note that the objective of problem (9) is only related to the total number of on-off switchings regardless of their specific locations (i.e., which EV and at which time). Thus, randomly allocating the limited charging capacity to competing EVs with the same EV specification will not affect the performance of problem (9). The basic idea of our algorithm is to iteratively linearize the quadratic objective function and efficiently search for a relatively smoother charging profile. Specifically, we replace the objective function of problem (9) with a linear function parameterized by the solutions from previous iterations,

$$G^l(\mathbf{U}, \mathbf{U}^{(k)}) = \sum_{n=1}^{N}\sum_{t=1}^{T}\alpha_{n,t}^{(k)}u_{n,t},$$

where $\alpha_{n,t}^{(k)} = \gamma_1 u_{n,t-1}^{(k)} + \gamma_2 u_{n,t}^{(k)} + \gamma_3 u_{n,t+1}^{(k)}$. Weighting factors γ_1, γ_2 and γ_3 are chosen to satisfy $\gamma_3 < \gamma_1 + \gamma_2$ (e.g., $\gamma_1 = -1, \gamma_2 = -6, \gamma_3 = -10$). Such weighting factors are valid because for any group of 1 with arbitrary length in vector u_n in each iteration, the weighting factor of the last 1 in each group is always larger than the weighting factor of the 0 just before this group, which indicates that this 1-0 pair will be inclined to exchange their location in the next iteration

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as long as the corresponding constraint is not violated. Thus, during each iteration, all the charging periods try to move one time slot ahead, which will eventually combine the charging periods together after the capacity constraints stop them from moving forward. Table I illustrates several iterations of the charging profile of EV n and its corresponding weighting factors, which validate the argument above. Moreover, such linearized objective is separable. Thus, the computation load for each iteration is equivalent to solving an LP.

We summarize our solution methodology for problems **OCDC** and **SOCDC** in Algorithm **SOCDC**. Note that our algorithm can achieve a rather smooth charging profile by solving successive LPs within a certain number of iterations, which is relatively efficient even when the input size is large.

V. Case Studies and Discussions

In this section, we evaluate the effectiveness of our proposed algorithm from multiple perspectives. For ease of illustration, we choose the flat load profile as our target load profile (i.e., $L(t) = 0, \forall t \in \mathcal{T}$) and quadratic function to penalize any deviation, which can result in a valley-filling profile. Our case studies consider a distribution grid with 100 residential buses and one commercial bus. Each residential bus has a capacity of 25 kVA and supports ten houses' electricity consumption while the commercial bus with 500 MVA capacity works for one public parking lot which can accommodate at most 400 EVs. To model EVs' uneven distribution over the residential buses, 10% and 90% of 800 EVs are randomly distributed in 20% and 60% of the residential buses respectively. The remaining 20% of the buses have no EVs that are charged at home. The hourly base load profile on each residential bus is randomly selected from the daily power consumption of a single home from July 1st to July 20th in the service area of the Southern California Edison [30] and is scaled by the number of houses attached to the bus (in our case, there are 10 houses attached to each bus). For the commercial bus, we assume there is no base load as it supports EV charging only. Based on [2], all the EVs are charged with single-phase level-2 charging rate of 3.3 kW.

A. Performance of **SOCDC** Algorithm

We first verify the valley-filling property of our proposed **SOCDC** algorithm. Without loss of generality, we assume that our scheduling horizon is from 18:00 to 08:00 on the next day, and during this period all the EVs are connected and require 6 hours to be fully charged. Fig. 5 shows the optimal aggregate load obtained from the **SOCDC** algorithm for different EV penetration levels. It can be shown that the aggregate load typically reveals a valley-filling property except when the EV penetration is too low (5%) or too high (100%). The reasons are as follows. In the low penetration case, there are not enough EVs to fill the load valley exactly even though all the EVs charge during the valley period. As for the high penetration case, due to the uneven EV distribution over buses, capacities of some buses are saturated while some other buses still possess additional capacities which cannot be further utilized. Therefore, the total load during valley periods is lower than the load during peak periods.

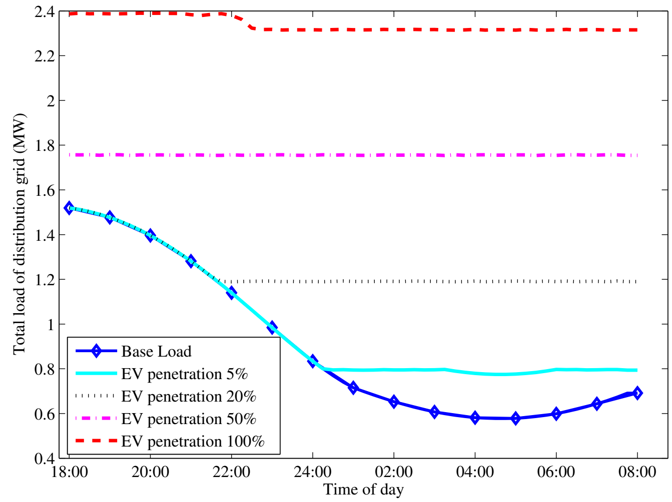

Fig. 5. Optimal aggregate load profiles for various EV penetrations. In the case of 100% EV penetration, there are totally 1200 EVs in the distribution grid in which 800 EVs charge at homes and 400 EVs charge at parking lots.

Though the **SOCDC** algorithm can smooth the total load profile effectively within the scheduling horizon, the distribution grid may encounter a deep load ramp at the end of the scheduling horizon (i.e., after 8 a.m.). In order to prevent this potential risk, we can reshape the total load profile by the modified objective function as follows

$$F(\mathbf{U}) = \sum_{t=1}^{T} w_t f_t(r_0 \sum_{n=1}^{N} u_{n,t} + D(t) - L(t)),$$

where w_t is the weighting factor at time t. Then, the following two modifications can be applied to prevent the risk of deep ramp: i) we choose a targeted load profile $L(t)$ with a moderate decreasing rate near the end of the time horizon. Then, by tracking $L(t)$, the final total load profile will not experience such a deep ramp. ii) If we keep $L(t) = 0$, we can modify the penalty functions $f_t(\cdot)$ or the weighting factors w_t for different t so that the penalty near the end of the time horizon is rather large. Hence, the modified objective function can force the EVs not to charge during the periods near the end of the time horizon. Fig. 6 illustrates the effectiveness of our methods to prevent the deep ramp risk. Here, we modify the objective function by changing the weighing factor w_t. In particular, we set $w_t = 1$ before 4 a.m. and w_t increase linearly with t after 4 a.m.. By adjusting the weighting factor w_t, we can achieve the load curves with different ramp rates at the end of the scheduling horizon.

To evaluate the advantages of the **SOCDC** algorithm, we compare the following charging schemes:

- **SOCDC:** Apply the **SOCDC** algorithm.
- **SOCDC-N:** Perform the **SOCDC** algorithm without capacity constraints (2).
- **Greedy:** Choose the EVs with larger energy demand to be charged first while avoid violating the capacity constraints.
- **Greedy-N:** Perform **Greedy** without capacity constraints.

Fig. 7 shows the total load profiles of different charging schemes. Compared with **Greedy** and **Greedy-N**, both **SOCDC** and **SOCDC-N** can achieve flatter total load profiles

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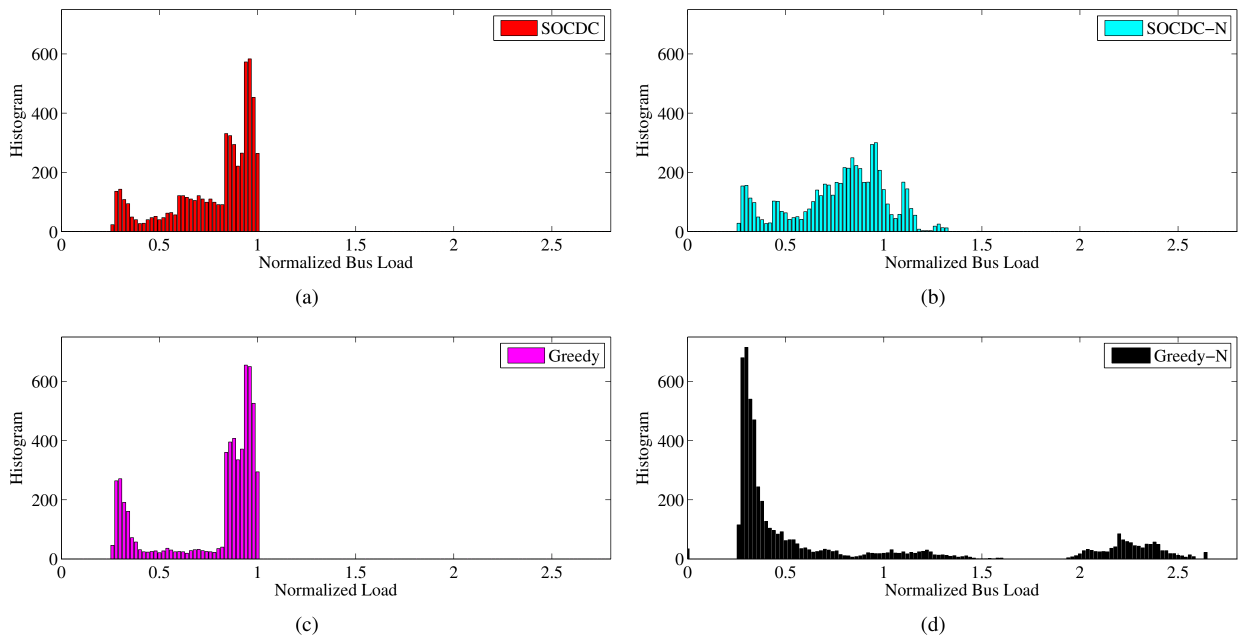

Fig. 8. Histogram frequency of normalized load on all the residential buses during the entire time horizon after applying different charging schemes. (a) **SOCDC**. (b) **SOCDC-N**. (c) **Greedy**. (d) **Greedy-N**.

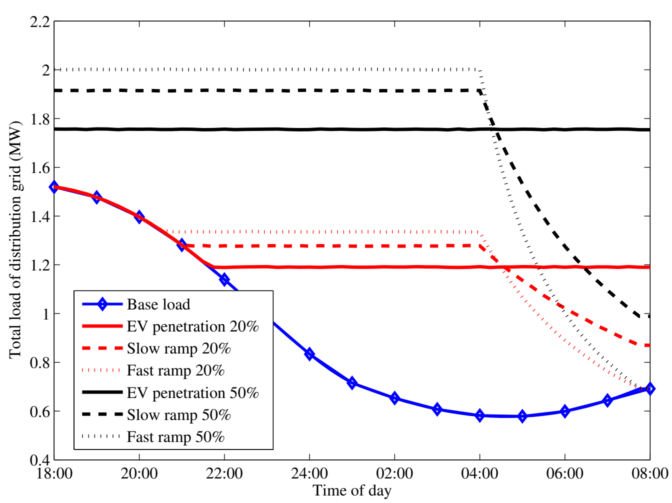

Fig. 6. Illustration of the effectiveness of our methods to prevent the deep ramp risk for different EV penetrations.

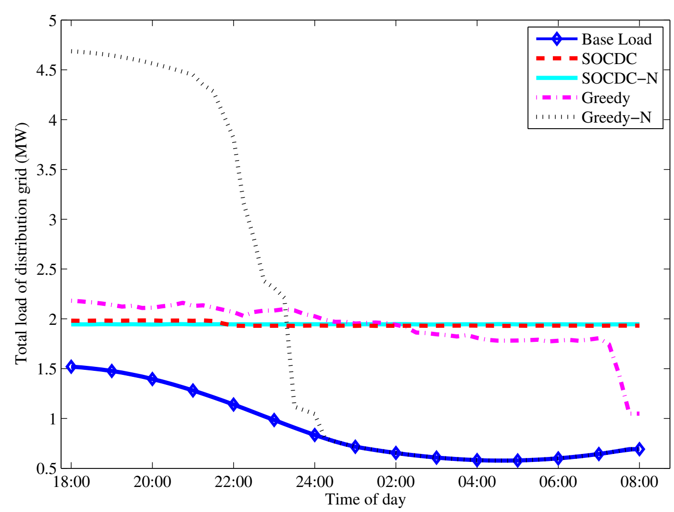

Fig. 7. Aggregate total load profiles for four scheduling schemes.

over the entire time horizon from 18:00 to 8:00. Moreover, **SOCDC-N** obtains a flatter profile in terms of the valley-filling behavior because the scheduling of **SOCDC** is restricted by the capacity constraints and thus less flexible in controlling the charging process. Fig. 8 depicts the distribution of the loads on all the residential buses over the entire time horizon. Each load sample represents the total load of one bus during one time slot and is normalized by the capacity of that bus. It is shown that when **SOCDC-N** and **Greedy-N** are applied, buses are possible to be overloaded, which threatens the stability of the distribution grid. In contrast, by applying **SOCDC**, the normalized load on each bus is less than or equal to 1. Therefore, **SOCDC** can strictly confine its load below the predetermined capacity of each individual bus all the time. Furthermore, compared with **Greedy**, **SOCDC** can reduce the number of heavily loaded buses.

Next, we show the performance of our proposed heuristic algorithm in minimizing the total number of on-off switchings. To test the robustness of our algorithm, we generate multiple scenarios for different EV penetrations. For each scenario, EV plug-in time varies uniformly from 18:00 to 20:00. Fig. 9 shows the number of on-off switchings of all the EVs before and after the smoothing procedure. In particular, the maximum, average and minimum of 100 simulations are illustrated for each EV penetration. Note that the benchmark we use is the total number of EVs participating in the charging scheduling, which is the loosest lower bound of problem

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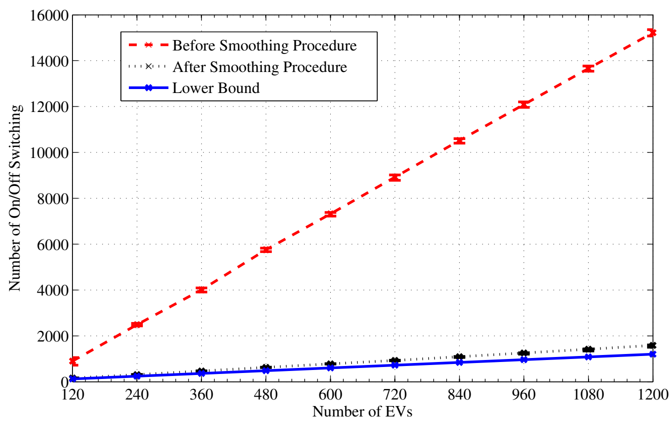

Fig. 9. Comparing the number of on-off switching before and after the smoothing procedure.

SOCDC. Thus, the real gap between our result and the optimal value is smaller than that shown in Fig. 9.

B. Illustration of Potential Real-time Implementations

Until now, we assume that we can predict the EV specifications exactly when applying the **SOCDC** algorithm. Towards a more realistic setting, we discuss the **SOCDC** algorithm's potential for real-time implementations, in which the prediction is unavailable or only available for a short time period.

Typically, the EV coordinator can obtain an EV's specification once the EV gets connected. In the worst case that the EV coordinator has no information about when EVs will get connected in the future, an intuitive way to schedule the EV charging process is to perform the **SOCDC** algorithm over all the connected EVs once a new EV gets connected. We name this incremental implementation by **ROCDC**. Figs. 10 (a) and (b) show the results of the offline **SOCDC** algorithm and the online **ROCDC** method under different plug-in time uncertainties and EV penetrations. We can observe that larger uncertainty of EV arrival or larger EV penetration results in larger deviation from the optimal solution when applying **ROCDC**. For the case with low EV penetration and small fluctuation in EV plug-in time, **ROCDC** can achieve results pretty close to the optimal solution as shown by the red curves in Fig. 10 (a).

ROCDC only considers the connected EVs. If the EV coordinator can predict the EV specifications precisely in the near future, the scheduling can be implemented in a way similar to the model predictive control. It works as follows: at time t, the EV coordinator estimates the information of EV arrivals within the time interval $[t, t+T_\text{P}]$ in addition to the known information of the connected EVs at time t, where T_P is the prediction time window. The **SOCDC** algorithm is performed for both connected EVs and estimated arriving EVs but only the scheduling solution at time t is applied. At time $t+1$, the previous procedure is repeated. We call this method **POCDC**. Fig. 10 (c) shows the performance of **POCDC**. The aggregate load is close to the optimal offline load profile with increase of the prediction window T_P. Thus, when the EV coordinator has the ability to predict EV information, **POCDC** can achieve a relatively satisfactory results for EV coordinated charging. Furthermore, we observe that the sub-optimality of the **POCDC** is mainly due to the underestimation of the EV load in the future. Thus, to improve the performance of the online algorithms, it is important to charge the connected EVs (i.e., the EVs that have already plugged in the power grid) more aggressively in case the unexpected EV load in the future will congest the power grid and lead to large load peaks.

VI. Conclusion

In this paper, we considered the EV coordinated discrete charging problem by taking into account the grid capacity constraints in distribution grid. The discrete optimization problem was formulated as two successive problems **OCDC** and **SOCDC**. Leveraging the properties of separable convex functions and total unimodularity, the problem **OCDC** was transformed into an equivalent LP, which could be solved efficiently and optimally. We further proved that the problem **SOCDC** is NP-hard and proposed a heuristic algorithm to minimize the on-off switchings for each EV's charging profile. Based on the simulation results, we demonstrated our algorithm's performance for various EV penetrations and observed the importance of grid capacity constraints compared with other scheduling methods. By applying our algorithm in incremental and prediction-based manners, we showed that our method could achieve reasonably good results under weak uncertainty or in the low EV penetration cases even without good predictions. Furthermore, connected EVs should be charged more aggressively to improve the performance of online algorithms in the cases with high EV arrival uncertainty or high EV penetration.

Appendix A
Proof of Theorem 1

Lemma 1. *An $I \times J$ matrix $\mathbf{A}$ is totally unimodular if and only if i) $\mathbf{A}$ has all its entries selected in $\{-1, 0, +1\}$ and ii) every row subset $\mathcal{I}$ can be divided into two disjoint sets, $\mathcal{I}_1$ and $\mathcal{I}_2$, such that $|\sum_{i\in\mathcal{I}_1} a_{ij} - \sum_{i\in\mathcal{I}_2} a_{ij}| \leq 1, \forall 1 \leq j \leq J$, where a_{ij} denotes the (i, j) element of matrix $\mathbf{A}$ [27].*

First of all, it is clear that all the entries of the coefficient matrix are either 0, 1 or -1. Thus, the first condition of Lemma 1 is satisfied. Note that the columns corresponding to variable v_t always satisfy the second condition of Lemma 1 no matter how the row subset is divided since for each column involved v_t, only one element equals to -1 and the rest are 0. Thus, it is sufficient to prove that for any row subset $\mathcal{I}$ of the coefficient matrix, it can be divided into two disjoint subsets, $\mathcal{I}_1$ and $\mathcal{I}_2$, and $|\sum_{i\in\mathcal{I}_1} a_{ij} - \sum_{i\in\mathcal{I}_2} a_{ij}| \leq 1, \forall j \in \mathcal{J}^{\mathbf{u}}$, where $\mathcal{J}^{\mathbf{u}}$ denotes the column subset associated with variables $u_{n,t}$.

Given $\mathcal{I}$, the way to construct $\mathcal{I}_1$ and $\mathcal{I}_2$ to satisfy Lemma 1 can be as follows. Let $\mathcal{I}^{(1)}$, $\mathcal{I}^{(2)}$ and $\mathcal{I}^{(5)}$ represent the subsets of $\mathcal{I}$ corresponding to constraints (1), (2) and (5), respectively.

Step 1. Group $\mathcal{I}^{(5)}$ into $\mathcal{I}_1$.

Step 2. If there exist rows in $\mathcal{I}^{(2)}$ that are associated with the same t as the rows in $\mathcal{I}^{(5)}$, group these rows into $\mathcal{I}_2$ and then group the rest of the rows of $\mathcal{I}^{(2)}$ into $\mathcal{I}_1$.

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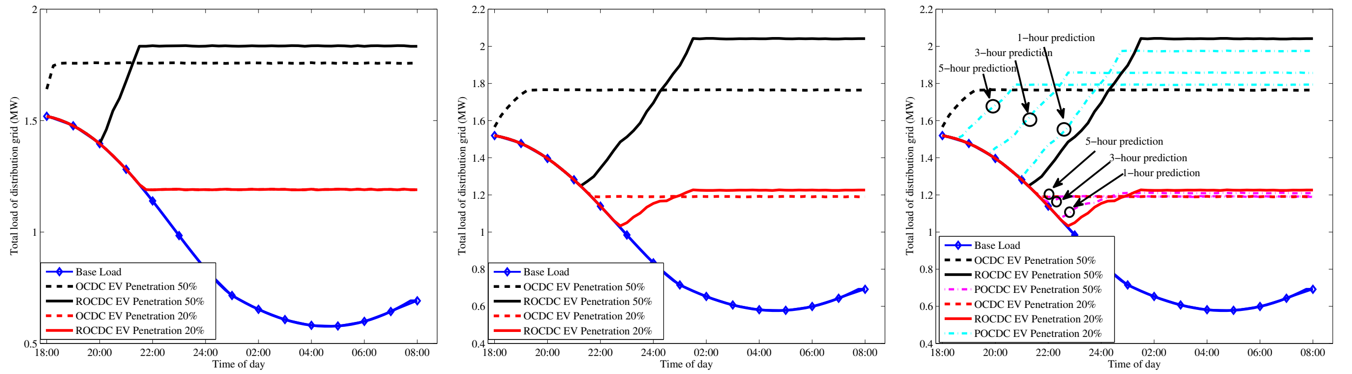

(a) (b) (c)

Fig. 10. Aggregate load profiles for **ROCDC** and **POCDC**. (a) **ROCDC** for 20% and 50% EV penetrations. Plug-in time is uniformly distributed from 18:00 to 22:00. (b) **ROCDC** for 20% and 50% EV penetrations. Plug-in time is uniformly distributed from 18:00 to 02:00 next day. (c) **POCDC** for 20% and 50% EV penetrations. Plug-in time is uniformly distributed from 18:00 to 02:00 next day.

Step 3. Group $\mathcal{I}^{(1)}$ into $\mathcal{I}_2$.

Each row in $\mathcal{I}^{(5)}$ has elements equal to 1 for the corresponding variable $u_{n,t}$ with a fixed t and $n \in \mathcal{N}$. After step 1, the summation of the rows in $\mathcal{I}_1$ is a row vector with all its entries being either 0 or 1. For the rows in $\mathcal{I}^{(2)}$, they only have elements equal to 1 for the corresponding variable $u_{n,t}$ with a fixed t and $n \in \mathcal{N}_m$. According to our grouping strategy in step 2, it can be guaranteed that $(\sum_{i\in\mathcal{I}_1} a_{ij} - \sum_{i\in\mathcal{I}_2} a_{ij}) \in \{0,1\}$, $\forall j \in \mathcal{J}^{\mathbf{u}}$. Since the sum of rows in $\mathcal{I}^{(1)}$ also contains either 0 or 1, after performing step 3, it is guaranteed that $(\sum_{i\in\mathcal{I}_1} a_{ij} - \sum_{i\in\mathcal{I}_2} a_{ij}) \in \{-1,0,1\}$, $\forall j \in \mathcal{J}^{\mathbf{u}}$. Thus, the coefficient matrix is proved to be totally unimodular.

Appendix B
Proof of Theorem 2

Problem (9) can be represented in matrix form as $\min_{\mathbf{u}}\{\frac{1}{2}\mathbf{u}'\mathbf{M}\mathbf{u} : \mathbf{A}\mathbf{u} \le \mathbf{b}, \mathbf{u} \in \{0,1\}^{NT}\}$, where $\mathbf{A}$ is totally unimodular and $\mathbf{b}$ is an integer vector. $\mathbf{M} = \mathbf{Q}'\mathbf{Q}$ is semidefinite, where $\mathbf{Q}$ is an N-block-diagonal matrix with each of its diagonal blocks a $T \times T$ matrix of the form

$$\begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & -1 \\ 0 & \dots & 0 & 0 & 1 \end{bmatrix}.$$

Therefore, $\mathbf{Q}$ is totally unimodular and non-singular. Based on Theorem 4.1 in [31], problem (9) is NP-hard in the strong sense.

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