A Novel Online Mechanism for Demand-Side Flexibility Management under Arbitrary Arrivals

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ABSTRACT

This paper proposes a price-based online mechanism for real-time demand-side flexibility management in smart grids. The major contribution of this paper is the design of a novel pricing function, based on which the price-based online mechanism achieves a tighter competitive ratio than previous work [3].

1 INTRODUCTION

Successful implementation of demand-side management programs necessitates a necessary collective behavior of power consumers with load flexibility. However, flexibility is a private resource owned by self-interested consumers (e.g, electric vehicle owners). Moreover, flexible loads are heterogeneous and can be controlled in multiple dimensions such as duration, rate/power and total amount/energy, etc. Therefore, without an appropriate *pricing and mechanism design*, consumers may not be well-incentivized to behave in a desired manner, leading to a poor system-wide performance.

In this paper, we propose a price-based online mechanism for real-time demand-side flexibility management. The major contribution of this paper is the design of a pricing function, based on which the price-based online mechanism achieves a bounded competitive ratio and improves the design in [3]. Meanwhile, our proposed mechanism is capable of eliciting multidimensional flexibility (including power-, energy-, and duration-flexibility) from consumers, and thus contributes to an efficient framework for real-time demand-side flexibility management in smart grids.

2 PROBLEM FORMULATION

We consider a group of agents $\mathcal{I} = \{1, \cdots, I\}$ with one flexibility aggregator over a slotted time horizon $\mathcal{T} = \{1, 2, \ldots, T\}$. Suppose each agent $i \in \mathcal{I}$ reports its consumption preference as a bid to the flexibility aggregator. We are interested in a dynamic scenario when agents arrive sequentially and there is no predictability in the sequence of agent arrivals, i.e., arbitrary arrivals. In such online settings, agents are sorted based on their reporting times (ties are broken arbitrarily). Once a new agent reports its bid, the mechanism needs to immediately perform the decision-making, including whether to accept this agent or not (i.e., bid selection), if yes then determines an irrevocable power schedule as well as the payment. The resulting *allocation rule* along with the *payment rule* constitutes an online mechanism [2].

The bid of agent i is described by $\boldsymbol{\theta}_i = (t_i, a_i, d_i, r_i, e_i, v_i)$, where t_i is the *reporting* time, a_i and d_i denote the *arrival* and *departure* times, r_i denotes the *maximum consumption rate*, and e_i denotes the *energy demand*. We use v_i to denote the monetary valuation of agent i for the bid, i.e., the maximum money agent i is willing to pay for receiving e_i units of energy during interval $\mathcal{T}_i \triangleq \{a_i, a_i+1, \ldots, d_i\}$. Note that $t_i \le a_i < d_i, \forall i \in \mathcal{I}$. In particular, when $t_i < a_i$, our bidding language captures cases with advance reservations. Our model allows such advance reservations, and thus is more general. We assume that the per-unit-energy valuation (unit: \$/kWh) of each agent is upper bounded by q, namely, $v_i/e_i \le q, \forall i \in \mathcal{I}$, where q can be interpreted as the maximum average bidding price. This assumption states that all the agents are rational and will not submit bids with exceptionally-high valuations.

Let us denote the status of bid $\boldsymbol{\theta}_i$ by binary variable $y_i \in \{0, 1\}$. Specifically, $y_i = 1$ denotes $\boldsymbol{\theta}_i$ is accepted and $y_i = 0$ otherwise. Note that we allow those rejected agents to submit a modified bid at a later time as new arrivals. Each bid $\boldsymbol{\theta}_i$ is associated with a candidate power schedule, denoted as $\boldsymbol{x}_i \triangleq \{x_i^t\}_{\forall t \in \mathcal{T}_i}$. According to the definitions of r_i and e_i, the power schedule $\boldsymbol{x}_i$ can be any vector in $\mathcal{X}_i$ defined as $\mathcal{X}_i \triangleq \left\{\boldsymbol{x}_i | \sum_{t\in\mathcal{T}_i} x_i^t = e_i y_i, x_i^t \in [0, r_i], \forall t \in \mathcal{T}_i\right\}$, where the first linear equality ensures e_i units of energy are allocated to agent i, and the second term comes from the definition of the maximum consumption rate.

We quantify the utility of agent i by a quasi-linear utility function $U_i = v_i y_i - \pi_i$, where π_i represents the payment charged by the aggregator. After processing all the agents, the utility of the aggregator is given by $U_a = \sum_{i\in\mathcal{I}} \pi_i - \sum_{t\in\mathcal{T}} f_t(w_b^t + \sum_{i\in\mathcal{I}} x_i^t)$, where w_b^t denotes the base load without flexibility, and $f_t(\cdot)$ denotes the cost function of power supply for the aggregator and is assumed to take the following quadratic form

$$f_t(\omega) = \begin{cases} \rho_2\omega^2 + \rho_1\omega + \rho_0 & \text{if } 0 \le \omega \le w_c^t, \\ +\infty & \text{if } \omega > w_c^t, \end{cases} \tag{1}$$

where w_c^t denotes the capacity limit at t, and ρ_2, ρ_1 and ρ_0 are given parameters. Meanwhile, we also assume that the base load w_b^t is known by the aggregator beforehand.

(**Offline Problem**) Summing over the utilities of all the agents and the flexibility aggregator leads to the social welfare of the whole system. Therefore, the offline social welfare maximization problem can be described as follows:

$$\max \quad \sum_{i\in\mathcal{I}} v_i y_i - \sum_{t\in\mathcal{T}} f_t\left(w_b^t + \sum_{i\in\mathcal{I}} x_i^t\right) \tag{2a}$$

$$s.t. \quad \boldsymbol{x}_i \in \mathcal{X}_i, y_i \in \{0,1\}, \forall i \in \mathcal{I}, \tag{2b}$$

$$\text{variables:} \quad \boldsymbol{x} = \{\boldsymbol{x}_i\}_{\forall i}, \boldsymbol{y} = \{y_i\}_{\forall i}. \tag{2c}$$

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Let $\text{SW}_{\texttt{opt}}$ denote the optimal objective value of the offline Problem (2), and let $\text{SW}_{\texttt{alg}}$ denote the social welfare achieved by an online algorithm/mechanism `alg`. The online mechanism `alg` is α-competitive if there exists a constant α such that $\text{SW}_{\texttt{alg}} \geq \frac{1}{\alpha}\text{SW}_{\texttt{opt}}$ holds for all possible arrival instances.

3 ONLINE MECHANISM DESIGN

We propose **PriMe** in Algorithm 1, a price-based online mechanism based on the online primal-dual approach [1]. At each round when there is a new arrival of agent $i \in \mathcal{I}$, the aggregator will decide whether to accept its bid or not. If the bid of agent i is accepted, the aggregator will then determine the corresponding power schedule $\boldsymbol{x}_i$ and the payment π_i. The same process will repeat upon the arrival of agent $i+1$.

(**Design Principle**) Let us denote the total power consumption by w_i^t *after* processing agent i. For notational convenience, let us define $w_0^t \triangleq w_b^t$, where w_0^t denotes the total power consumption before accepting the first agent. Therefore, we have $w_i^t = w_{i-1}^t + x_i^t y_i$. Intuitively, we have $w_i^t = w_{i-1}^t$ if agent i is rejected, namely $y_i = 0$. Let us denote the marginal power price *after* processing agent i by λ_i^t, and suppose we have a pricing function $\phi_t(\omega)$, which updates the marginal power price λ_i^t based on the incumbent power consumption w_i^t, namely, $\lambda_i^t = \phi_t(w_i^t)$. Following the above definition, the initial marginal power price, namely the price before accepting the first agent, is given by $\lambda_0^t = \phi_t(w_0^t) = \phi_t(w_b^t)$. Suppose we can *design such a pricing function ϕ_t that depends on the current total power consumption, then the entire decision-making processes of* **PriMe** *rely on causal information only, which thus facilitates an online implementation.*

(**Pricing Function**) Based on the above principle, we present our design of $\phi_t(\omega)$ as a two-segment function as follows:

$$\phi_t(\omega) = \begin{cases} 4\rho_2(\omega - w_b^t) + p_b^t, & \text{if } \omega \in [w_b^t, w_{\text{thr}}^t), \\ m_t \exp\left(\frac{\alpha_t \omega}{w_c^t - w_b^t}\right) + f_t'(\omega) + \frac{p_c^t - p_b^t}{\alpha_t}, & \text{if } \omega \in [w_{\text{thr}}^t, w_c^t], \end{cases}$$

where $w_{\text{thr}}^t = (w_c^t + w_b^t)/2$, $p_b^t = f_t'(w_b^t)$, and $p_c^t = f_t'(w_c^t)$. As shown in Fig. 1, w_{thr}^t denotes the threshold such that $\phi_t(w_{\text{thr}}^t) = p_c^t$. Meanwhile, $\phi_t(w_c^t) = q$ always holds. Therefore, the two unknown parameters in $\phi_t(\omega)$, namely, m_t and α_t, can be obtained by solving the following system of two equations:

$$\begin{cases} m_t \exp\left(\frac{\alpha_t w_{\text{thr}}^t}{w_c^t - w_b^t}\right) + \frac{p_c^t - p_b^t}{\alpha_t} = p_c^t - f_t'(w_{\text{thr}}^t), \\ m_t \exp\left(\frac{\alpha_t w_c^t}{w_c^t - w_b^t}\right) + \frac{p_c^t - p_b^t}{\alpha_t} = q - p_c^t. \end{cases} \tag{3}$$

(**Competitive Ratio**) **PriMe** achieves a tighter competitive ratio than [3]. The details are summarized in the following theorem.

THEOREM 3.1. *Let us define $q_{\text{thr}}^t \triangleq p_c^t + \frac{\exp(2)+1}{4}(p_c^t - p_b^t), \forall t \in \mathcal{T}$, and further define $\mathcal{T}_s$ as a subset of $\mathcal{T}$ such that $q_{\text{thr}}^t < q, \forall t \in \mathcal{T}_s$, then the competitive ratio of* **PriMe** *is given as follows:*

- *If $\mathcal{T}_s = \emptyset$, i.e., $q_{\text{thr}}^t \geq q, \forall t \in \mathcal{T}$,* **PriMe** *is 4-competitive.*
- *If $\mathcal{T}_s \neq \emptyset$, i.e., $q_{\text{thr}}^t < q, \exists t \in \mathcal{T}$,* **PriMe** *is $\max_{t\in\mathcal{T}_s}\{\alpha_t\}$-competitive, where $\alpha_t > 4$ is the unique solution to the following equation:*

$$\frac{\alpha_t \xi_t - 2}{\alpha_t - 2} = \exp\left(\frac{\alpha_t}{2}\right), \forall t \in \mathcal{T}_s, \tag{4}$$

where $\xi_t \triangleq \frac{2(q-p_c^t)}{p_c^t - p_b^t}$ and $\xi_t > \frac{\exp(2)+1}{4}$ ≈ 2.097.

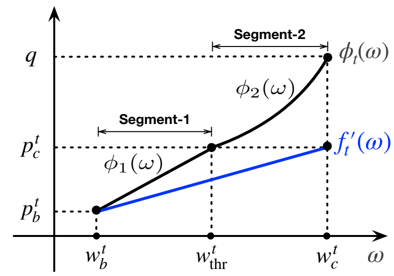

Figure 1: Illustration of the proposed pricing function $\phi_t(\omega)$ and the marginal cost function $f_t'(\omega) = 2\rho_2\omega + \rho_1$. Note that we implicitly assume that q is larger than p_c^t for all $t \in \mathcal{T}$.

Algorithm 1: Pricing Mechanism (**PriMe**)

1: Initialize $w_0^t = w_b^t, \lambda_0^t = \phi_t(w_0^t), \forall t$.
2: **while** a new agent i reports **do**
3: Collect the bid $\boldsymbol{\theta}_i$ from agent i;
4: Calculate the candidate $\{x_{i,*}^t\}_{\forall t \in \mathcal{T}_i}$ for agent i based on

$$\min \quad \sum\nolimits_{t=\mathcal{T}_i} x_i^t \lambda_{i-1}^t \tag{5a}$$
$$s.t. \quad \sum\nolimits_{t=\mathcal{T}_i} x_i^t \geq e_i, \qquad (\mu_i) \tag{5b}$$
$$0 \leq x_i^t \leq r_i, \forall t \in \mathcal{T}_i, \tag{5c}$$

5: Set the marginal energy price μ_i by

$$\mu_i = \max_{t\in\mathcal{T}_i, x_i^t > 0} \lambda_{i-1}^t.$$

6: Determine the utility of agent i by

$$U_i = v_i - \mu_i e_i.$$

7: **if** $U_i < 0$ **then**
8: Reject agent i (i.e., set $y_i = 0$).
9: **else**
10: Accept agent i (i.e., set $y_i = 1$).
11: Determine the power schedule as $\{x_{i,*}^t\}_{\forall t \in \mathcal{T}_i}$.
12: Determine the payment as $\pi_i = \mu_i e_i$.
13: Update the total load: $w_i^t = w_{i-1}^t + x_{i,*}^t, \forall t$.
14: Update the marginal power price: $\lambda_i^t = \phi_t(w_i^t), \forall t$.
15: **end if**
16: **end while**

4 CONCLUSION

In this paper, we proposed **PriMe**, a price-based online mechanism for real-time demand-side flexibility management in smart grids. **PriMe** is capable of eliciting multidimensional flexibility from power consumers, and achieves a tighter competitive ratio than the existing work [3].

REFERENCES

[1] Niv Buchbinder and Joseph (Seffi) Naor. 2009. The Design of Competitive Online Algorithms via a Primal-Dual Approach. *Found. Trends Theor. Comput. Sci.* 3, 2-3 (Feb. 2009), 93–263.
[2] David C. Parkes. 2007. *Online Mechanisms.* Cambridge University Press, Chapter 16, 411–439.
[3] B. Sun, X. Tan, and D.H.K. Tsang. 2018. Eliciting multi-dimensional flexibilities from electric vehicles: a mechanism design approach. *IEEE Transactions on Power Systems* (2018). in press.