Eliciting Multi-Dimensional Flexibilities From Electric Vehicles: A Mechanism Design Approach

Bo Sun, *Student Member, IEEE*, Xiaoqi Tan, *Member, IEEE*, and Danny H. K. Tsang, *Fellow, IEEE*

***Abstract*—Electric vehicles (EVs) have been well recognized as a deferrable load with the flexibility to shift their energy demands over time. Although this one-dimensional flexibility has been extensively investigated both by research and industrial implementations, the expanding energy demand and the associated uncertainties still make the integration of a large population of EVs into power system reliably and economically greatly challenging. In this paper, we design an auction scheme via mechanism design to elicit two additional flexibilities from EVs, namely energy flexibility and deadline flexibility. An offline mechanism is firstly designed as a benchmark based on the famous Vickrey–Clark–Groves mechanism. Then based on the primal-dual approach, we propose an online auction, in which all bids are truthful, the loss of social welfare is bounded by competitive ratio, and the mechanism can be implemented in polynomial time. By the numerical results, we quantitatively show that both the power system operators and individual EVs can benefit from the integration of the multi-dimensional flexibilities through our proposed mechanisms.**

***Index Terms*—Electric vehicle, multi-dimensional flexibilities, mechanism design.**

I. Introduction

ONE distinguished feature of future smart grids is the transformation of demand-side electricity loads from dumb followers into proactive participators. Leveraging the flexibilities in reducing or delaying the electricity consumptions, various conventional loads are capable of actively responding to the regulation signals from power systems and making profits in electricity markets [1]. Specifically, the charging control of electric vehicles (EVs) has been considered as a key component in the demand-side management programs. The well-recognized flexibility from EVs is their deferrable property, namely, the ability to shift their fixed energy demand over their plug-in time horizon. Although such one-dimensional flexibility has been extensively investigated by the state-of-the-art research [2]–[5], the expanding energy demand and associated uncertainties from unpredictable human behaviors still introduce great challenges to integrate a large population of EVs into power systems reliably and economically [6]. Therefore, other dimensional flexibilities are becoming increasingly important for better integration of EVs into power systems. In this paper, we aim at identifying and exploiting two novel and promising flexibilities beyond the deferrable property of EVs: the *energy-flexibility* and the *deadline-flexibility*. To be more specific, recent advances in the EV charging infrastructure have greatly relieved the range anxiety from the energy-critical EV owners and hence relaxed their energy demands during a single charging period [7]. Therefore, EVs are able to take advantage of the energy-flexibility by adjusting their energy demands for the sake of economic benefits. Besides, depending on EVs' different patterns of use (e.g., commuting, taxi, etc.) and EV drivers' preferences, some EVs are not sensitive to the charging deadlines, and hence are capable of delaying their charging requests up to some extended deadlines [8]. Such deadline-flexibility can significantly facilitate the operators to maintain the reliability and reduce the energy cost during the peak hours of power systems.

In the current setting of EV coordinated charging problems [2]–[5], the charging requests are purely decided by EVs based on the electricity tariff (e.g., time-of-use tariff) information to maximize their own utility (i.e., valuation minus energy cost). Whereas the system operators consider the charging requests just as fixed parameters, and determine the charging profiles of EVs to optimize their own objectives (e.g., minimizing the load variance [2], [3] or minimizing energy cost [4], [5]). Therefore, the system operator fails to provide proper incentives to encourage EVs' participation, and the flexibilities inside the charging requests are not taken into account. To integrate the aforementioned multi-dimensional flexibility[1] (MDF) in the EV coordinated charging problem, *we propose to capture the energy-flexibility and deadline-flexibility by modeling the valuations[2] of each EV's charging request (i.e., energy demand, arrival time and deadline) as a function of the required energy demand and deadline*. For example, a short-commute EV can describe its

[1]The multi-dimensional flexibility refers to the flexibilities in adjusting the total amount and the charging time duration of energy, which are considered as different types of traded commodities in the proposed mechanism.

[2]The valuation in this paper refers to the maximal money that an EV is willing to pay for different energy demands and deadlines.

Manuscript received September 1, 2017; revised March 15, 2018; accepted July 9, 2018. Date of publication July 16, 2018; date of current version August 22, 2019. This work was supported by the Hong Kong Research Grants Councils General Research Fund through Project no. 16209814 and Project no. 16210215. Paper no. TPWRS-01355-2017. *(Corresponding author: Bo Sun.)*

B. Sun and D. H. K. Tsang are with the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong (e-mail: bsunaa@ust.hk; eetsang@ust.hk).

X. Tan is with the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON M5S 2E4, Canada (e-mail: xiaoqi.tan@utoronto.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRS.2018.2856283

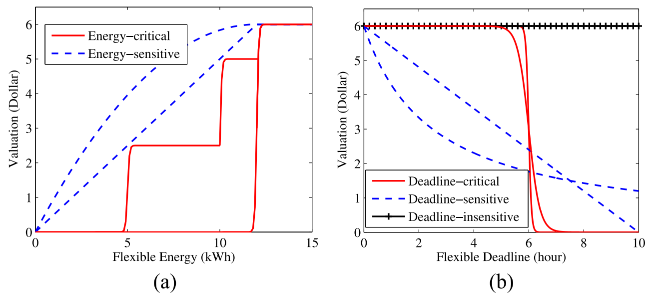

Fig. 1. Illustration of valuation functions of the flexible energy and deadline. (a) Energy-flexibility. (b) Deadline-flexibility.

energy-critical charging request by specifying a valuation function that steps into a higher value only after a certain amount of energy is satisfied (e.g., solid-red curves in Fig. 1(a)). Such functions are also referred to as all-or-nothing utility functions in [8]. Differently, for EV taxis, their valuations are sensitive to the energy but not critically dependent on it. Then, they can describe their valuations by non-decreasing functions (e.g., blue-dashed curves in Fig. 1(a)). As for the deadline-flexibility, an EV with a stringent deadline can describe its valuation as a deadline-critical function that decays rapidly to zero after its target deadline (e.g., solid-red curves in Fig. 1(b)). On the other hand, EVs with a flexible time schedule can represent their valuations as functions that gradually decrease with their deadlines (e.g., dashed-blue curves in Fig. 1(b)). In the extreme case that EVs are totally insensitive to the deadlines, the valuation functions are just flat lines (e.g., the black curve in Fig. 1(b)).

Based on the above modeling of energy and deadline flexibilities, we can proceed to integrate these flexibilities into the original EV coordinated charging problems and design a win-win mechanism for both the system operator and individual EVs. Specifically, we aim at designing mechanisms that can elicit the MDF to improve the social welfare of the EV coordinated charging problems, and quantifying the benefits of these flexibilities in the charging requests for both system operators and EVs. Particularly, this work makes contributions in the following aspects: (i) We propose a general formulation for the social welfare problem of the EV coordinated charging, in which the valuation is a function of both energy demand and deadlines, and the charging cost is a general convex function with capacity constraints. (ii) We adopt a Vickrey–Clark–Groves (VCG)-based mechanism in the offline setting as a benchmark to quantify the benefits of utilizing MDFs for both the power system and EV owners. (iii) To handle online EV arrivals, an online mechanism is designed to ensure the truthfulness of the auction scheme and bound the loss of the social welfare due to the lack of future EV arrival information in the online setting.

II. Related Works

In the literature, extensive research has been carried out on the EV coordinated charging problems to mitigate negative impacts [2], [3] or gain profits [4], [5] leveraging the EVs' deferrable property. However, the energy-flexibility and deadline-flexibility are ignored by these previous works. The pioneering work [9] studies the energy-flexibility in a generic demand response problem and it elicits this flexibility by an auction approach based on the famous VCG mechanism. In parallel, some works also examine the deadline-flexibility in the deferrable load such as EVs. The work [10] first addresses the value of the deadline-flexibility from EVs for solar energy integration. However, [10] fails to provide insights into the exploitation of deadline-flexibility. [8] proposes to elicit the deadline-flexibility by a pricing approach, in which a deadline differentiated pricing contract is designed and analyzed. [11] further proposes to utilize the MDF, and numerically validates their benefits in power systems. However, the above auction and pricing approaches are designed for offline problems, in which all EVs need to participate in the mechanisms at the beginning of the decision horizon. While in practice, EVs often arrive at the charging facilities one by one with random inter-arrival time, and the system operator only has the information from EVs that are currently connected to the charging facilities. Therefore, the mechanism is more likely to be implemented in an online manner for the EV coordinated charging problem. To handle this online setting, [12] proposes an online mechanism by assuming that all EVs can discharge energy back to the power grid, and prove the truthfulness of the bids from EVs. The follow-up work [13] further takes into account the generation cost with renewable energy, and characterizes a general class of truthful online mechanisms. However, both works [12], [13] have no guarantee on the loss of the social welfare due to lacking the information of EV arrivals in the online setting. Based on [12] and [13], the work [14] designs an online mechanism for the coordinated EV charging problem with the consideration of energy cost. Moreover, based on the primal-dual analysis, the proposed mechanism shows the competitive ratio of the online algorithm to bound the loss of social welfare.

Our paper is most relevant to work [14]. In particular, we both formulate the objective in the same form, namely, the total valuations of all EVs minus the electricity cost, and apply the primal-dual approach to analyze the competitive ratio of the online mechanisms. Compared to work [14], we specifically make contributions in three aspects: (i) We adopt a more general problem formulation, in which the valuation of EVs is a function of not only the total energy amount but also the deadline, that makes it possible to trade MDFs in the mechanism. (ii) We propose to reduce the loss due to the online arrivals of bids by redesigning the marginal electricity cost function so that charging capacity can be reserved for late-coming bids. This design principle is different from [14] which reschedules the charging profiles of all accepted EVs in a receding horizon manner. (iii) Our mechanism can ensure that EVs submit their truthful bids to the system operator, which makes it possible for practical implementation when EVs have their own private preferences.

In the following, we start by formulating an offline social welfare maximization problem and introducing a VCG-based mechanism as a benchmark. Then we propose an online mechanism, which not only guarantees the truthfulness of EVs' bids, but also bounds the loss of the social welfare due to the online arrivals of bids. We assume the online mechanism has no information about the bid arrival process in the future. The same assumption has been adopted by most of the prior works [4], [12]–[14] when studying the online mechanisms. Although the future bid arrival process may be predicted with certain accuracy in practice, we postpone the online mechanism design with predicted information for our future research.

III. Social Welfare Problem and Offline Mechanism Design Benchmark

We start by formulating the social welfare maximization (SWM) problem for the EV coordinated charging problem, based on which both offline and online mechanisms will be designed in the following sections. We consider the scenario that an aggregator (or system operator) coordinates and schedules the charging processes of a population of EVs to optimize social objectives over a fixed time horizon. Let $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ and $t \in \mathcal{T} = \{1, 2, \ldots, T\}$ be the sets of EVs and time slots, respectively. In order to elicit the MDF from the submitted charging requests of EVs, we define the interaction between the aggregator and the EVs as an auction scheme: (i) Each EV n submits its charging bid to the aggregator. Each bid is a vector of quadruples $\boldsymbol{\theta}_n = \{\boldsymbol{\theta}_{n,k}\}_{k\in\mathcal{K}_n} = \{e_{n,k}, a_{n,k}, d_{n,k}, b_{n,k}\}_{k\in\mathcal{K}_n}$, where $e_{n,k}$, $a_{n,k}$, and $d_{n,k}$ denote the energy demand, arrival time, and deadline of EV n, $b_{n,k}$ is the corresponding claimed valuation for $e_{n,k}$, $a_{n,k}$, and $d_{n,k}$, and $\mathcal{K}_n$ denotes the set of bids from EV n. (ii) Based on the reported bids $\boldsymbol{\Theta} = \{\boldsymbol{\theta}_n\}_{n\in\mathcal{N}}$ from all EVs, the aggregator determines the winning bid selection $\boldsymbol{y}_n = \{y_{n,k}\}_{k\in\mathcal{K}_n}$, charging schedule $\boldsymbol{x}_n = \{x^t_{n,k}\}_{\{k\in\mathcal{K}_n, t\in\mathcal{T}\}}$ and payment p_n for each EV n so that the social objective can be optimized. Particularly, the winning bid selection $y_{n,k}$ equals 1 if bid k of EV n is selected and 0 otherwise. The charging schedule $x^t_{n,k}$ denotes the transferred energy into EV n within time slot t if bid k is selected.

A. Social Welfare Maximization Problem

We formally formulate the SWM problem by integrating the MDF into the EV coordinated charging problem as follows.

$$\max_{\boldsymbol{x},\boldsymbol{y}} \quad \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}_n} b_{n,k}y_{n,k} - \sum_{t\in\mathcal{T}} c_t\left(\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}_n} x^t_{n,k}\right), \tag{1a}$$

$$\text{s.t.} \quad \sum_{t\in\mathcal{T}_{n,k}} x^t_{n,k} = e_{n,k}y_{n,k}, \quad n\in\mathcal{N}, k\in\mathcal{K}_n, \tag{1b}$$

$$\sum_{k\in\mathcal{K}_n} y_{n,k} \le 1, \quad n\in\mathcal{N}, \tag{1c}$$

$$y_{n,k}\in\{0,1\}, \quad n\in\mathcal{N}, k\in\mathcal{K}_n, \tag{1d}$$

$$0\le x^t_{n,k}\le X_n, \quad n\in\mathcal{N}, k\in\mathcal{K}_n, t\in\mathcal{T}. \tag{1e}$$

The first term in the objective (1a) is the total valuations of all EVs and the second term represents the cost of the aggregator. The cost function $c_t(\cdot)$ for time slot t is assumed to be a general convex function, which can take different forms for different purposes (e.g., load valley-filling, energy cost minimization) of the aggregator. Energy constraints (1b) guarantee that each EV is charged the energy based on the selected bid. The time slot set $\mathcal{T}_{n,k} = \{a_{n,k}, \ldots, d_{n,k}\}$ represents the plug-in time period for bid k of EV n. Constraints (1c) and (1d) specify the feasible set of the winning bid selection $\boldsymbol{y} = \{\boldsymbol{y}_n\}_{n\in\mathcal{N}}$, and ensure that each EV can win no more than one bid. Constraints (1e) restrict the charged energy per time slot by the maximum charging rate of each EV. X_n denotes the maximum amount of energy that can be transferred by EV n in each time slot. Let F^* and $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ denote the optimal objective value and solution of the SWM problem, respectively.

The SWM problem can be solved by either the aggregator or an independent platform, and the social welfare can be maximized by implementing the optimal solution $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ under the condition that all the bids from EVs truthfully reveal their valuations on their charging requests. However, it is often not the case without proper incentives. For self-interested EVs, their goals are to maximize their own utility $u_n = v_n(\boldsymbol{x}_n, \boldsymbol{y}_n) - p_n$, where $v_n(\cdot)$ is the true valuation function of EV n, which may differ from their claimed valuations. Hence EVs may not reveal their true valuations if they can obtain a higher utility when they submit an alternative bid. When the EVs misreport their valuations, the aggregator may lose social benefits when optimizing the social objectives based on the reported bids. In order to ensure the truthfulness of submitted bids, we propose to elicit the MDF by a mechanism design approach. In particular, we aim at designing the mechanisms for determining the winning bid selection, charging schedule and payment in the defined auction between the aggregator and EVs, so that the following three goals can be achieved: *i*) **Truthfulness:** Each EV maximizes its own utility when it reveals its true valuation regardless of others' bids; *ii*) **Economic efficiency:** the social welfare is maximized; and *iii*) **Computational efficiency:** The mechanism can be implemented in polynomial time. In the rest of this section, we consider an offline setting as a benchmark, where all the bids are assumed to be collected at the beginning of the time horizon. By this setting, the contributions of the MDF to the social welfare can be quantified.

B. Offline Auction by VCG Mechanism

As the milestone in the mechanism design field, the VCG mechanism provides a systematic way to ensure the truthfulness and economic efficiency for the SWM problem. Specifically, the VCG mechanism states: *i*) The winning bid selection and charging schedule are the optimal solution of the SWM problem (i.e., $\boldsymbol{x}^*$ and $\boldsymbol{y}^*$); *ii*) the payment is determined by

$$p_n = h_n(\boldsymbol{\theta}_{-n}) - \left[\sum_{m\in\mathcal{N}_{-n}}\sum_{k\in\mathcal{K}_m} b_{m,k}y^*_{m,k} - \sum_{t\in\mathcal{T}} c_t\left(\sum_{m\in\mathcal{N}}\sum_{k\in\mathcal{K}_m} x^{t*}_{m,k}\right)\right], \forall n\in\mathcal{N}, \tag{2}$$

where $\boldsymbol{\theta}_{-n}$ denotes all the reported bids excluding EV n and $h_n(\boldsymbol{\theta}_{-n})$ is an arbitrary function depending on $\boldsymbol{\theta}_{-n}$. In particular, we can choose $h_n(\boldsymbol{\theta}_{-n})$ as the optimal solution of the SWM problem when EV n is excluded from the system. Different from the classical VCG mechanisms, in which the objective of the SWM problem is only the summation of all the valuations (i.e., the first term of the objective (1a)), our problem is the welfare maximization with energy costs [9], [14]. The validation of the VCG payment can be verified following the same proof as Proposition 3 in [9].

The SWM problem is a mixed-integer non-linear program, which is known to have computational complexity growing exponentially in the number of integer optimization variables [22]. Though heuristic or approximation algorithms [15] can be applied to solve the SWM problem in polynomial time, truthfulness is no longer necessarily guaranteed by substituting the sub-optimal solutions into the VCG payment (2) [16]. Worse still, although the VCG mechanism can handle the offline scenario, EVs often arrive at the charging systems and join the auction in an online manner in practice. Thus, once a new EV submits its bid, the aggregator needs to immediately determine whether to accept one of its bids, and decides its charging schedule as well as its corresponding payment if any bid is accepted. In this online setting, applying the VCG mechanism to the existing EVs in the system cannot guarantee a satisfactory performance because *i*) each time an EV arrives, the mechanism needs to be invoked once and the resulting computational complexity is rather high; *ii*) aggregator may allocate too much cheap energy to the low-value bids that come early but have to reject high-value bids later due to the limited amount of cheap energy. Thus, we will continue to discuses this more challenging online mechanism design problem for practical systems in the following section.

IV. Online Mechanism via Primal-Dual Approach

The online mechanism design is more challenging because the mechanism not only needs to ensure the truthfulness, and computational efficiency, the loss of the social welfare due to the online nature has to be bounded. Our goal is to design an online mechanism whose resulting social welfare is competitive with respective to the optimal offline solution. An online algorithm is α-competitive for some $\alpha \geq 1$ if it achieves at least $1/\alpha$ of the optimal offline social welfare in the worse case. In this work, we will quantify the loss of the social welfare by the competitive ratio.

A. Design Principle of the Online Primal-Dual Approach

Our online mechanism bounds the loss of the social welfare based on the primal-dual approach [14], [17]. We firstly relax the integer variable $\boldsymbol{y}$ in the SWM problem to be continuous, and obtain the primal problem of the relaxed SWM problem:

$$\max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{v}} \quad \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}_n} b_{n,k}y_{n,k} - \sum_{t\in\mathcal{T}} c_t\left(v_t\right) \tag{3a}$$

$$\text{s.t.} \quad v_t \geq \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}_n} x_{n,k}^t, \quad t\in\mathcal{T}, \tag{3b}$$

$$\sum_{t\in\mathcal{T}_{n,k}} x_{n,k}^t \geq e_{n,k}y_{n,k}, \quad n\in\mathcal{N}, k\in\mathcal{K}_n, \tag{3c}$$

$$\sum_{k\in\mathcal{K}_n} y_{n,k} \leq 1, \quad n\in\mathcal{N}, \tag{3d}$$

$$x_{n,k}^t \leq X_n, \quad n\in\mathcal{N}, k\in\mathcal{K}_n, t\in\mathcal{T}, \tag{3e}$$

$$\boldsymbol{x},\boldsymbol{y},\boldsymbol{v} \geq \boldsymbol{0}, \tag{3f}$$

where the slack variable v_t denotes the total load at time t. Because the cost function $c_t(v_t)$ is non-decreasing in v_t, the inequality constraints (3b) and (3c) will be binding at the optimal solution as the case in [14]. Therefore, the relaxation of the primal problem only lies in variables $\boldsymbol{y}$. Let $\boldsymbol{\lambda} = \{\lambda_t\}_{\{t\in\mathcal{T}\}}$, $\boldsymbol{\mu} = \{\mu_{n,k}\}_{\{n\in\mathcal{N},k\in\mathcal{K}_n\}}$, $\boldsymbol{\eta} = \{\eta_n\}_{\{n\in\mathcal{N}\}}$, and $\boldsymbol{\beta} = \{\beta_{n,k}^t\}_{\{n\in\mathcal{N},k\in\mathcal{K}_n,t\in\mathcal{T}\}}$ be the dual variables corresponding to constraints (3b)–(3e). Then the dual problem of the relaxed SWM problem can be described as

$$\min_{\substack{\boldsymbol{\lambda},\boldsymbol{\mu},\\ \boldsymbol{\eta},\boldsymbol{\beta}}} \quad \sum_{n\in\mathcal{N}}\eta_n + \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}_n}\sum_{t\in\mathcal{T}}\beta_{n,k}^t X_n + \sum_{t\in\mathcal{T}}\hat{c}_t(\lambda_t) \tag{4a}$$

$$\text{s.t} \quad \eta_n \geq b_{n,k} - \mu_{n,k}e_{n,k}, \quad n\in\mathcal{N}, k\in\mathcal{K}_n, \tag{4b}$$

$$\lambda_t \geq \mu_{n,k} - \beta_{n,k}^t, \quad n\in\mathcal{N}, k\in\mathcal{K}_n, t\in\mathcal{T}, \tag{4c}$$

$$\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\eta},\boldsymbol{\beta} \geq \boldsymbol{0}, \tag{4d}$$

where $\hat{c}_t(\lambda_t) = \max_{v_t\geq 0}\left(\lambda_t v_t - c_t(v_t)\right)$ is the conjugate function of the cost function $c_t(v_t)$.

Next, we design the online auction algorithm to elicit the MDF in the EV coordinated charging problem based on the primal and dual problems (3) and (4). Let P_n and D_n be the objective values of the primal and dual problems after processing the EV n's bid. Then P_N and D_N are the final primal and dual objective values of the online algorithm. The basic idea of the primal-dual approach is to design an online algorithm to determine the feasible primal and dual variables for the problems (3) and (4) so that the increment of the primal objective is no smaller than that of the dual objective by a constant factor α after processing each EV's bid. Then the competitive ratio of the online algorithm can be determined by the following lemma.

Lemma 1: If there exists a constant $\alpha \geq 1$ such that $P_n - P_{n-1} \geq \frac{1}{\alpha}(D_n - D_{n-1})$, $\forall n$, and the initial dual objective $D_0 = 0$, the online algorithm is α-competitive.

Proof: Firstly, we have $P_0 = 0$, and hence $P_N = \sum_{n=1}^{N}(P_n - P_{n-1}) \geq \frac{1}{\alpha}\sum_{n=1}^{N}(D_n - D_{n-1}) = \frac{1}{\alpha}D_N$. Recall that we denote the maximal social welfare by F^*. Let F_r^* be the optimal value of the relaxed primal problem (3). Then, we have $F_r^* \geq F^*$. Due to the weak duality [21], $D_N \geq F_r^* \geq F^*$. Thus, the online algorithm is α-competitive. □

B. Online Auction by Primal-Dual Approach

Following the principle of the primal-dual approach, we start by designing an online mechanism to solve the SWM problem with a guaranteed competitive ratio, and then prove that the online mechanism is truthful and computationally efficient. In this mechanism, upon the arrival of EV n, it submits its bids to the aggregator, and then the aggregator decides whether to accept one of its bids, and how to schedule its charging profile based on the selected bid. EV n is informed of the decision from the aggregator immediately. If one bid is accepted, its associated charging schedule and payment will not be changed, and EV n is not allowed to withdraw the accepted bid. If all bids are rejected, EV n needs to adjust its bidding price $b_{n,k}$ or charging request $\{e_{n,k}, a_{n,k}, d_{n,k}\}$, and resubmit the modified bids to the aggregator for getting dispatched. Next, we show the design details on the determination of the bid selection, the charging schedule and the payment in the online mechanism. From the aggregator's perspective, it only has the system-level knowledge of the per-slot total load of the previous $n-1$ EVs (i.e., v_t^{n-1}) and their corresponding per-slot energy price (i.e., λ_t^{n-1}). After receiving the bid $\boldsymbol{\theta}_n$ from EV n, the aggregator firstly determines the candidate charging profile $\boldsymbol{x}_{n,k} = \{x_{n,k}^t\}_{t\in\mathcal{T}_{n,k}}$ that satisfies the energy constraints (3c) in the primal problem for each bid $k \in \mathcal{K}_n$. In particular, $\boldsymbol{x}_{n,k}$ is achieved by solving a simple cost minimization problem based on the per-slot energy price λ_t^{n-1}.

$$\begin{aligned} \boldsymbol{x}_{n,k} = \underset{0\le x_{n,k}^t \le X_n}{\arg\min} \quad & \sum_{t\in\mathcal{T}_{n,k}} x_{n,k}^t \lambda_t^{n-1}, \\ \text{s.t.} \quad & \sum_{t\in\mathcal{T}_{n,k}} x_{n,k}^t \ge e_{n,k}. \end{aligned} \tag{5}$$

The optimal solution of the problem (5) can be simply derived by allocating the energy demand to the cheapest time slots with the restriction of per-slot transferred energy capacity X_n. Then, based on $\boldsymbol{x}_{n,k}$, we can obtain the corresponding marginal energy price $\mu_{n,k}$ in the dual problem, namely, the price of the next unit of energy for bid k, as follows,

$$\mu_{n,k} = \max\nolimits_{t:x_{n,t}^k>0} \lambda_t^{n-1}. \tag{6}$$

According to the form of the energy cost in the right hand side (RHS) of constraints (4b), we propose to use $\mu_{n,k}$ as the energy price of EV n. Therefore, when bid k is selected, the utility of EV n can be calculated by $b_{n,k} - \mu_{n,k}e_{n,k}$, which is exactly the RHS of of constraints (4b). Then, the next question is whether to accept one bid and which one to choose if one bid is accepted. From the complementary slackness in the KKT conditions [21] of constraints (4b), $y_{n,k}$ must be zero unless $\eta_n = b_{n,k} - \mu_{n,k}e_{n,k}$. To ensure that EV n has the incentive to participate in the auction, we must guarantee a non-negative utility for EV n. Thus, we propose to accept the bid that maximizes the utility of EV n as long as the maximal utility is non-negative. In particular, we let the utility of EV n be

$$\eta_n = \left[\max_{k\in\mathcal{K}_n} b_{n,k} - \mu_{n,k}e_{n,k}\right]^+. \tag{7}$$

Accordingly, we determine the bid selection $y_{n,k}$ by

$$y_{n,k} = \begin{cases} 1 & \text{if } \eta_n > 0 \text{ and } k = k', \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

where $k' = \arg\max_{k\in\mathcal{K}_n} b_{n,k} - \mu_{n,k}e_{n,k}$ is the index of the bid that maximizes the utility of EV n. Once bid k' is accepted, its corresponding charging schedule $\boldsymbol{x}_{n,k'}$ and payment $p_n = \mu_{n,k'}e_{n,k'}$ are both determined.

Next, based on the complementary slackness of constraints (4c), we determine the dual variable $\beta_{n,k}^t$ by

$$\beta_{n,k}^t = \begin{cases} \mu_{n,k} - \lambda_t^{n-1} & x_{n,k}^t > 0, \\ 0 & x_{n,k}^t = 0. \end{cases} \tag{9}$$

Finally, we update the system-level variables, namely the per-slot total load

$$v_t^n = v_t^{n-1} + x_{n,k'}^t, \quad t \in \mathcal{T}_{n,k'}, \tag{10}$$

and the per-slot energy price

$$\lambda_t^n = f_t(v_t^n), \quad t \in \mathcal{T}_{n,k'}, \tag{11}$$

where $f_t(\cdot)$ is a specifically-designed function to update the energy price appropriately. Note that an intuitive method to design the energy price function is setting it to be the marginal energy cost at the current total load of time t, namely, $f_t(v_t^n) = c_t'(v_t^n)$. However, because the bids arrive in an online manner, the marginal cost pricing method may lead to aggressive acceptance of the bids that come early but are with low valuation over energy, which may result in severe loss of the social welfare. For example, one possible scenario is that EVs with low-value bids participate in the mechanism early and the limited amount of energy capacity is all allocated to these EVs. Later, a large number of EVs with high-value bids come. However, they will be rejected due to insufficient energy capacity for charging schedule. Therefore, although the marginal cost pricing method can reflect the true electricity cost for serving the current EV, it makes the mechanism accept the early-coming bids too aggressively. Thus, we need to design the pricing function $f_t(\cdot)$ properly to reserve some capacity for the late-coming bids. Particularly, the pricing function $f_t(\cdot)$ needs to be specifically-designed for different forms of the cost functions $c_t(\cdot)$. In this paper, we mainly show the results when $c_t(\cdot)$ is a convex quadratic cost function with a capacity limit W_t for each time slot t, namely,

$$c_t(v_t) = \begin{cases} b_t v_t + a_t v_t^2 & v_t \le W_t, \\ +\infty & v_t > W_t. \end{cases} \tag{12}$$

Based on $c_t(v_t)$, we propose to adopt a speed-up version of the marginal pricing function

$$\lambda_t = f_t(v_t) = \begin{cases} c_t'(\delta v_t) & v_t \le W_t/\delta, \\ c_t'(W_t)e^{\xi(v_t - W_t/\delta)} & W_t/\delta < v_t \le W_t. \end{cases} \tag{13}$$

where $\delta = 2$ and

$$\xi = \max\left\{\frac{2\ln(\frac{U}{c_t'(W_t)})}{W_t}, \frac{2a_t}{b_t + a_t W_t}\right\}$$

are design parameters and the determination of these two parameters are shown in the proof of Theorem 1 in Appendix A. Before stepping into the details of determining the parameters, we firstly explain the key idea for the design of this pricing function. When the total load is not very large (i.e., $v_t \le W_t/\delta$), a speed-up scaler δ is used to artificially boost the marginal cost of energy in case the cheap energy is used up by the early arrivals with low-value bids. As the total load approaches the capacity (i.e., $W_t/\delta < v_t \le W_t$), the marginal cost of energy increases exponentially fast. In addition, the energy price when $v_t = W_t$ is guaranteed to be larger than the maximum per-unit valuation in all bids, which is denoted by U when determining the parameter ξ. Based on this setting, the charging profile that may lead to exceeding the capacity W_t will not be selected because in that case, $\mu_{n,k} > U$ leads to a negative-utility bid, which will be rejected by the aggregator. Note that U is unknown to the aggregator before all bids have arrived. Thus, U needs to be estimated based on the historical data of the bids. In the numerical section, we will show that a reasonably good performance can be achieved even when the estimated value largely deviates from U.

Finally, the online mechanism is summarized in Algorithm 1. Based on the design principle in the primal-dual approach, Algorithm 1 guarantees a bounded social welfare as shown in Theorem 1.

Theorem 1: The online mechanism Algorithm 1 is α-competitive in social welfare for

$$\alpha = \max\left\{\frac{2(b_t + 2a_t W_t)\ln(\frac{U}{c_t'(W_t)})}{a_t W_t}, 4\right\}.$$

Algorithm 1: Proposed Online Mechanism.

1: **Initialization:** Collect the energy cost function $c_t(\cdot)$, $\forall t \in \mathcal{T}$; set the per-slot total load $v_t^0 = 0, \forall t \in \mathcal{T}$ and the per-slot energy price $\lambda_t^0 = c_t'(0), \forall t \in \mathcal{T}$.
2: **while** a new EV n arrives **do**
3: Collect the bid $\boldsymbol{\theta}_n$ from EV n;
4: **for** $k \in \mathcal{K}_n$ **do**
5: Calculate the charging schedule of bid k as $\boldsymbol{x}_{n,k}$ based on (5);
6: Calculate the marginal energy price $\mu_{n,k}$ by (6);
7: **end for**
8: Determine the bid that maximizes EV n's utility by $k' = \arg\max_{k \in \mathcal{K}_n} b_{n,k} - \mu_{n,k} e_{n,k}$, and choose the utility of EV n to be $\eta_n = b_{n,k'} - \mu_{n,k'} e_{n,k'}$;
9: **if** $\eta_n < 0$ **then**
10: Reject EV n;
11: **else**
12: Accept bid k';
13: Determine the charging schedule as $\boldsymbol{x}_{n,k'}$;
14: Determine the payment as $p_n = \mu_{n,k'} e_{n,k'}$;
15: Update v_t^n and λ_t^n by (10) and (11) ;
16: **end if**
17: **end while**

Proof: Please refer to Appendix A. □

Algorithm 1 not only achieves a bounded competitive ratio as we design, but also ensures the truthfulness and computational efficiency, which are necessary conditions for implementation of the online mechanism in practice.

Theorem 2: The online mechanism Algorithm 1 is truthful and runs in polynomial time.

Proof: (Truthfulness) We first prove the truthfulness in the bidding prices (or valuations) in EVs' bids. To achieve this goal, we firstly show that Algorithm 1 can ensure that (i) EVs cannot affect the potential payment of all their bids by manipulating their bidding prices; (ii) the bid that can maximize the utility of each EV is always chosen. The second argument follows directly from the design of Algorithm 1 as shown in (7) and (8). As for the first argument, recall that the bid k from EV n can be divided into two parts: the charging request $\{e_{n,k}, a_{n,k}, d_{n,k}\}$ and the associated bidding price $b_{n,k}$. Algorithm 1 determines the potential payment of the bid k from EV n as $\mu_{n,k} e_{n,k}$. From (6) and (13), $\mu_{n,k}$ only depends on the charging request $\{e_{n,k}, a_{n,k}, d_{n,k}\}$ and the charging schedules of the first $n-1$ EVs. Notice that the charging schedules from the first $n-1$ EVs are not allowed to reschedule in Algorithm 1. Therefore, the potential payment of bid k from EV n is independent of its bidding price, which equivalently means that EV n cannot manipulate its potential payment for each bid k by misreporting its bidding price $b_{n,k}$. After verifying the arguments (i) and (ii), we can continue to show that EV n cannot improve its utility by misreporting its bidding prices. Suppose EV n submits its truthful bidding prices, Algorithm 1 selects the bid for EV n by $k' = \arg\max_{k \in \mathcal{K}_n} b_{n,k} - \mu_{n,k} e_{n,k}$ if $b_{n,k} - \mu_{n,k} e_{n,k} \geq 0$. This selection method is aligned with the goal of EV n and EV n's utility is maximized. Suppose EV n misreports its bidding prices as $\hat{b}_{n,k}$, we show that misreporting bidding prices cannot improve its utility but is at risk of losing utility. There are two possibilities if EV n misreports its bidding prices: (i) the selected bid $\hat{k}'$ based on the misreported $\hat{b}_{n,k}$ is the same as k'. Then, the true utility of EV n is not improved. (ii) the selected bid $\hat{k}'$ is different from k'. Then the bid selection is sub-optimal for maximizing the utility of EV n, and thus the utility of EV n decreases. In summary, the utility of EV n in the online mechanism cannot be improved by misreporting its bidding prices. Submitting the truthful bidding prices is a dominant strategy for each EV.

Next, we proceed to prove that EV n is truthful in reporting its arrival time $a_{n,k}$ and deadline $d_{n,k}$ in the online mechanism. The true arrival time is defined as the earliest time epoch that an EV can physically reach the charging station when it submits the bids. Similarly, the true deadline is defined as the latest time epoch that an EV can keep plugging-in before it leaves. We make the natural assumption that EVs cannot submit the bids that claim an arrival time that is earlier than its true arrival time, or a deadline that is later than its true departure time [12], [13]. This assumption rules out the bids in which EVs cannot follow what they claim. Note that the marginal energy price $\mu_{n,k}$ in the potential payment of bid k from EV n is non-increasing in the plug-in duration of the EV n (i.e., $d_{n,k} - a_{n,k}$). Thus, EV n cannot reduce its potential payment by reporting a late arrival (i.e., a larger $a_{n,k}$) or early departure (i.e., a smaller $d_{n,k}$). Submitting the truthful arrival time and deadlines is a dominant strategy for each EV.

(Polynomial Running Time) To process the bid of EV n, it firstly takes $O(KTlog(T))$ time to compute the charging schedule and marginal price for each bid (i.e., lines 4–7), where $K = \max_n |\mathcal{K}_n|$. This is because the optimal solution of the problem (5) can be computed by allocating X_n units of energy to the $\lfloor e_{n,k}/X_n \rfloor$ time slots with the smallest per-slot prices, and the remaining energy to the $\lfloor e_{n,k}/X_n \rfloor + 1$ smallest time slot. This is equivalent to the computation complexity of a sorting algorithm, which is just $O(Tlog(T))$. Then, the bid selection (i.e., line 8) can be computed in $O(K)$. Next we can decide whether to accept the bid (i.e., line 9) in a constant time. If the bid is accepted, the corresponding charging schedule and payment (i.e., 13–14) can be also computed in a constant time, and per-slot total load and per-slot price (i.e., lines 15) can be updated in $O(T)$ time. Summarizing the above results, Algorithm 1 runs in $O(NKTlog(T))$. □

V. Case Studies

In this section, we numerically evaluate the performance of the proposed offline and online mechanisms for different application scenarios. In both settings, we examine the resulting social objectives and the bid selection, from which we demonstrate the performance improvement for both the aggregator and individual EVs due to the integration of energy-flexibility and deadline-flexibility in the EV coordinated charging problem. The duration of time slot in the following numerical tests is set to be 15 minutes and all the EVs are charged with the single-phase level-2 charging rate 3.3 kW. Thus the transferred energy per time slot is limited by $X_n = 0.825$ kWh, $n \in \mathcal{N}$.

TABLE I
CANDIDATE BIDS FOR THE VALLEY-FILLING SCENARIO

$\vartheta_1 = \{20\text{kWh}, 18, 06, \$4.2\}$	$\vartheta_2 = \{20\text{kWh}, 18, 09, \$4\}$
$\vartheta_3 = \{16\text{kWh}, 18, 06, \$3.68\}$	$\vartheta_4 = \{16\text{kWh}, 18, 09, \$3.52\}$
$\vartheta_5 = \{12\text{kWh}, 18, 06, \$3\}$	$\vartheta_6 = \{12\text{kWh}, 18, 09, \$2.88\}$

A. *Offline Mechanism for Valley-Filling*

We first consider a scenario that the aggregator elicits the MDF to achieve a valley-filling load profile. In this case, a total of 200 EVs participate in the auction mechanism for overnight charging in a residential area. All the EVs are required to submit their bids before 18:00 and hence the offline mechanism is applicable to this scenario. To achieve the valley-filling profile, which has been studied by the previous work [2], we define the cost function of the aggregator as $c_t(v_t) = \omega(v_t + D(t))^2$, where ω is the cost factor transferring the squared load into a monetary value and $D(t)$ is the base load at time t. The base load profile $\{D(t)\}_{t\in\mathcal{T}}$ is obtained by scaling the daily power consumption of homes in the service area of the Southern California Edison [18]. As the increase of ω, the valley-filling load profile is more desired by the aggregator. Table I lists six candidate bids that each EV may submit. The valuations in these candidate bids are set based on the principle that the per-unit energy valuation is decreasing in both the energy demand and deadlines because the urgency of energy decreases as the increase of the energy demand and the convenience of EVs also decreases when their deadlines are extended. In order to quantitatively show the benefits from the energy-flexibility and deadline-flexibility, we compare four different cases, in which EVs submit bids with different levels of flexibilities.

- **Benchmark (BEN):** Deadline and energy flexibilities are not considered in the system, so each EV only submits its largest energy demand with the earliest deadline to selfishly maximize its own utility, namely, $\boldsymbol{\theta}_n = \{\vartheta_1\}$.
- **Deadline-flexibility (DF):** Each EV can postpone its charging deadline for a cheaper per-unit energy price. Each EV offers its deadline-flexibility by submitting bid $\boldsymbol{\theta}_n = \{\vartheta_1, \vartheta_2\}$.
- **Energy-flexibility (EF):** Each EV can reduce its energy demand to a lower amount with a higher per-unit energy valuation to avoid bid rejection. Each EV sets $\boldsymbol{\theta}_n = \{\vartheta_1, \vartheta_3, \vartheta_5\}$.
- **Deadline-flexibility and Energy-flexibility (DF-EF):** Both the deadline-flexibility and the energy-flexibility are considered. Each EV submits the bid $\boldsymbol{\theta}_n = \{\vartheta_k\}_{k=1,\ldots,6}$.

The valley-filling profiles resulting from different cases are compared in Fig. 2. We can observe that the total load profiles are much flatter if EVs can offer their deadline-flexibility and/or energy-flexibility. Particularly, the deadline-flexibility enables the aggregator to shift the EV charging load to extended time slots, which not only reduces the maximum total load before 06:00 but also helps avoid the sharp drop in the total load around 06:00. In addition, the energy-flexibility makes it possible for the aggregator to choose some bids with lower energy demand so that more EVs can be served instead of rejecting EVs to maintain a flatter load profile. This point can be observed clearly in Table II, which shows the ratio of the accepted bids to total

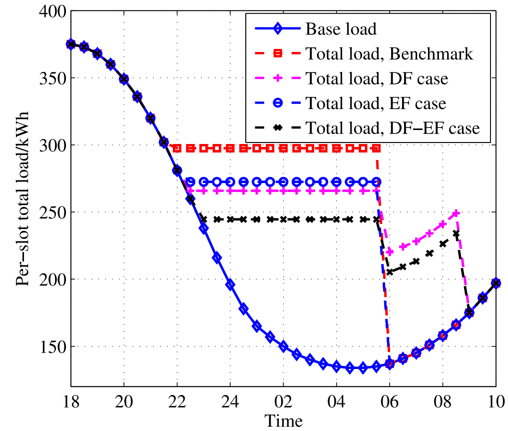

Fig. 2. Illustrating the valley-filling profile with different levels of flexibilities.

TABLE II
RATIO OF ACCEPTED BIDS TO TOTAL NUMBER OF EVS IN THE VALLEY-FILLING SCENARIO WITH DIFFERENT COST FACTORS $\omega \times 10^{-4}$

	ω	ϑ_1	ϑ_2	ϑ_3	ϑ_4	ϑ_5	ϑ_6	All
BEN	3	1	–	–	–	–	–	1
	4	0.73	–	–	–	–	–	0.73
	5	0.38	–	–	–	–	–	0.38
DF	3	0.52	0.48	–	–	–	–	1
	4	0.49	0.5	–	–	–	–	0.99
	5	0.25	0.24	–	–	–	–	0.49
EF	3	0	–	1	–	0	–	1
	4	0	–	0	–	1	–	1
	5	0	–	0	–	1	–	1
DF-EF	3	0	0	0.61	0.39	0	0	1
	4	0	0	0	0	0.68	0.32	1
	5	0	0	0	0	0.66	0.34	1

number of EVs with the increase of cost factor under different levels of flexibilities. From the composition of the accepted bids, we can see that the bids with lower energy demands and longer deadlines are more likely to be chosen with the increase of cost factor. Based on this mechanism, the aggregator can elicit more flexibilities from EVs by increasing its cost factor so that a flatter load profile can be achieved to maintain the stability of the power system. From the perspective of EVs, for the same cost factor, the ratio of accepted bids is much higher when more flexibilities are offered. For instance, the cases with energy-flexibility (i.e., EF and DF-EF) accept at least one bid for each EV while the other cases reject bids from some EVs when the cost factors are large.

B. *Online Mechanism for Energy Cost Minimization*

This section considers an online EV charging scenario, in which the aggregator controls the EV charging processes for a charging station and aims at minimizing the total energy cost. We consider a time-invariant electricity cost function which is defined in (12) $\forall t \in \mathcal{T}$. Therefore, we can omit the time index of the parameters and set the cost factors $b = 10^{-4}$ \$/kWh, $a = [1.6, 3.2, 4.8, 6.4, 8] \times 10^{-4}$ \$/kWh/kW and the capacity $W = 300$ kW based on the parameter settings in [4]. The EVs arrive at the charging station in an online manner and the aggregator has no information about the future EV arrivals. The basic charging requests in our test follow the data set that is developed in [19] based on the real traces of taxi vehicles. From this

TABLE III
SUBMITTED BIDS OF EV n IN COST MINIMIZATION SCENARIO

$\vartheta_1 = \{e_n, a_n, d_n^e, U_1 e_n\}$	$\vartheta_2 = \{e_n, a_n, d_n, U_2 e_n\}$
$\vartheta_3 = \{\gamma_1 e_n, a_n, d_n^e, \gamma_1 U_3 e_n\}$	$\vartheta_4 = \{\gamma_1 e_n, a_n, d_n, \gamma_1 U_4 e_n\}$
$\vartheta_5 = \{\gamma_2 e_n, a_n, d_n^e, \gamma_2 U_5 e_n\}$	$\vartheta_6 = \{\gamma_2 e_n, a_n, d_n, \gamma_2 U_6 e_n\}$

data set, we have the arrival time a_n, deadline d_n and energy demand e_n for each EV n. According to these basic charging requests, each EV can submit six bids to the aggregator as shown in Table III. In these bids, the deadline-flexibility is offered by submitting the earliest deadline $d_n^e = \lceil e_n/X_n \rceil$ or the true deadline d_n for each EV. The energy-flexibility is provided by reducing its energy demand at a discount of $\gamma_1 = 0.8$ or $\gamma_2 = 0.6$. U_k represents the per-unit energy valuation for each bid k. To distinguish the high-value bids (HB) and low-value bids (LB), we choose two different settings of U_k, namely the LB with $\{U_k\}_k = \{0.3, 0.2, 0.4, 0.3, 0.5, 0.4\}$ \$/kWh and the HB with $\{U_k\}_k = \{0.5, 0.4, 0.6, 0.5, 0.7, 0.6\}$ \$/kWh. Based on these settings, our proposed online mechanism (PM) Algorithm 1 is undertaken and its performance is compared with other benchmark algorithms as follows.

- **Optimal offline benchmark (OPT):** optimal solution of the SWM problem assuming all the bids of EVs are known at the beginning of the time horizon.
- **Online mechanism with myopic price (MM):** this mechanism is the same as Algorithm 1 except the update function of the per-slot energy price in line 15 of Algorithm 1. MM updates λ_t^n by the per-slot marginal energy price, namely $\lambda_t^n = c_t'(v_t^n)$. Therefore, MM myopically maximizes the social welfare based on current price information, which may lead to the rejection of high-value bids that arrives later.
- **Online greedy mechanism (GM):** this mechanism greedily allocates per-slot energy to bid with the highest valuation until the energy capacity is used up.

Fig. 3 illustrates the total energy cost and the competitive ratio of different algorithms in the online settings of EV coordinated charging problem. It can be observed from Fig. 3(a) that PM can effectively minimize the total energy cost compared to other algorithms including the optimal social welfare solution OPT. It means PM allocates the energy more conservatively to avoid the potential congestion of the energy demand and its resulting high electricity cost in this online setting. For both MM and GM, its allocation of energy is more aggressive to satisfy the current EV charging demand and hence may lead to sever high cost if the electricity cost increases rapidly with the total load (i.e., the increase of cost factor b). In Fig. 3(b), the competitive ratios of the three online algorithms are compared. We find that our proposed algorithm PM can achieve a competitive ratio close to 1 although the theoretical value is more than 4 as shown in Theorem 1. Both MM and GM perform worse than PM in terms of the competitive ratio in our real-trace test, and have no theoretical bounds to guarantee the worst case performance. Moreover, in the three online algorithms, only PM and MM can be proved to guarantee the truthfulness of the mechanism because they both belong to the posted pricing mechanisms as shown in the proof of Theorem 2. We can observe that PM outperforms MM

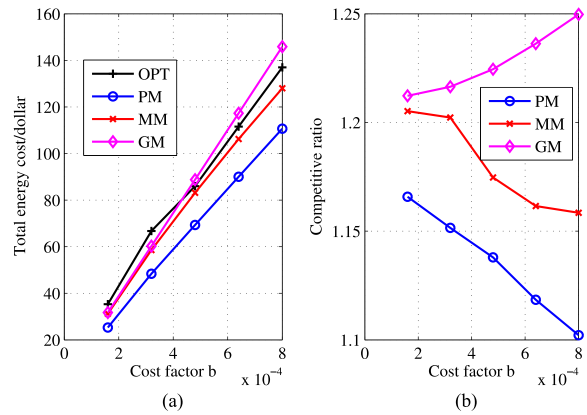

Fig. 3. Comparison of PM with other benchmark mechanisms. (a) Total energy cost. (b) Competitive ratio.

TABLE IV
RATIO OF ACCEPTED BIDS TO TOTAL NUMBER OF EVS IN THE COST MINIMIZATION SCENARIO FOR BOTH HB AND LB

		ϑ_1	ϑ_2	ϑ_3	ϑ_4	ϑ_5	ϑ_6	All
OPT	LB	0	0	0.13	0	0.19	0.02	0.34
	HB	0.01	0	0.56	0.03	0.06	0.01	0.66
	All	0.01	0	0.69	0.03	0.24	0.03	1
PM	LB	0	0	0.23	0	0.11	0	0.34
	HB	0	0	0.19	0.03	0.31	0.03	0.56
	All	0	0	0.41	0.03	0.42	0.03	0.9
MM	LB	0	0	0.33	0	0.01	0	0.34
	HB	0.07	0	0.29	0.03	0.08	0.04	0.52
	All	0.07	0	0.62	0.03	0.09	0.04	0.85
GM	LB	0	0	0.34	0	0	0	0.34
	HB	0.25	0.01	0.04	0	0.13	0.05	0.48
	All	0.25	0.01	0.38	0	0.13	0.05	0.81

in both cost minimization and competitive ratio from our test cases. In addition, in Table. IV, we show the ratio of the accepted bids from a total of 500 EVs for both LB and HB. We find that the acceptance ratios of the three online algorithms are not as high as that of OPT due to lacking the information of future EV arrivals. Although the three online algorithms can achieve the same acceptance ratio for LB, PM outperforms MM and GM in the acceptance ratio of HB. Thus, our proposed mechanism PM can effectively prohibit the rejection of the HB that may arrive late at the charging stations. Therefore, our design of the update function in (13) is validated to be effective to benefit both the aggregator and individual EVs.

Because the aggregator cannot know the exact value of the maximum per-unit valuation (i.e., U) before all bids have arrived, the aggregator needs to estimate U as the input of the online mechanism. Let U^e denote an estimation of U. When U^e is used as the input of the online mechanism, we investigate its impact on the total energy cost and the competitive ratio of the online mechanism. In Fig. 1, we compare fives cases in which U can be underestimated or overestimated by varying the ratio U^e/U, from 0.5 to 1.5. It can be observed that even when the estimated value U^e largely deviates from U, the resulting total energy costs and competitive ratios in different cases are relatively close to each other. In Fig. 4(a), the total energy cost roughly reflects the total number of accepted bids in different cases. We can find that with the increase of the ratio U^e/U, the mechanism becomes more conservative, which results in accepting less bids and hence leads to a smaller total energy

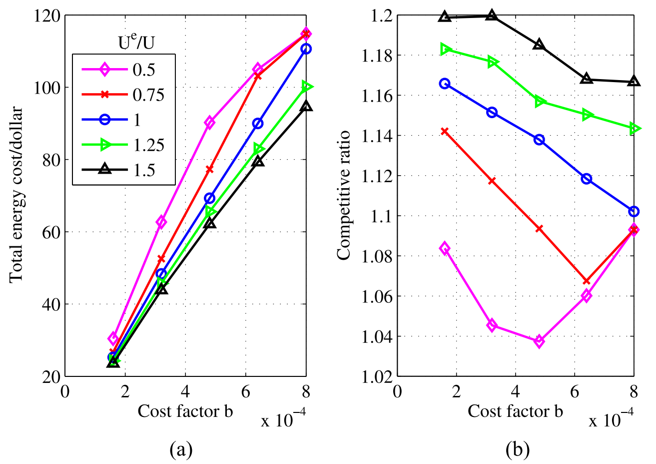

Fig. 4. Illustrating the impact of the estimated maximum per-unit valuation. Different marked curves show the results with different ratios U^e/U. (a) Total energy cost. (b) Competitive ratio.

cost. In Fig. 4(b), we can observe that an overestimation of U leads to a larger competitive ratio. This result can be expected based on the theoretic bound

$$\alpha = \max\left\{\frac{2(b_t + 2a_t W_t)\ln(\frac{U}{c_t'(W_t)})}{a_t W_t}, 4\right\},$$

which is non-decreasing in U. However, with the particular EV traces in this numerical test, an underestimation of U can even achieve a better competitive ratio compared to the exact value.

VI. Conclusion

In this paper, we have proposed an online mechanism to help integrate the MDF, namely, the deferrable property, the energy-flexibility, and the deadline-flexibility into the EV coordinated charging problems. We start our mechanism design by proposing a social welfare maximization problem. Based on this social problem, a VCG-based offline mechanism has been designed to elicit the MDF truthfully with the maximal social welfare as a benchmark. Furthermore, in the online setting, we have proposed an online auction, in which all bids are truthful, the loss of social welfare is bounded by competitive ratio, and the mechanism can be implemented in polynomial time. Our numerical results have shown an evident performance improvement for both the system operator and the EVs from the integration of multi-dimensional flexibilities.

Appendix A
Proof of Theorem 1

Based on the design of the online auction, it is clear that the primal variables $(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{v})$, and the dual variables $(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}, \boldsymbol{\beta})$ from Algorithm 1 are feasible for the primal and dual problems, respectively. Thus, our goal is to prove that $P_n - P_{n-1} \geq \frac{1}{\alpha}(D_n - D_{n-1})$ and find the minimal α. If EV n is rejected, the inequality holds because $P_n - P_{n-1} = D_n - D_{n-1} = 0$. Otherwise,

$$\begin{aligned}
P_n - P_{n-1} &= b_{n,k'} - \sum_{t\in\mathcal{T}}\left[c_t(v_t^n) - c_t(v_t^{n-1})\right] \\
&= \eta_n + \mu_{n,k'}e_{n,k'} - \sum_{t\in\mathcal{T}}\left[c_t(v_t^n) - c_t(v_t^{n-1})\right] \\
&= \eta_n + \sum_{t\in\mathcal{T}_{n,k'}}\left(\lambda_t^{n-1} + \beta_{n,k'}^t\right)\left(v_t^n - v_t^{n-1}\right) \\
&\quad - \sum_{t\in\mathcal{T}}\left[c_t(v_t^n) - c_t(v_t^{n-1})\right] \\
&= \eta_n + \sum_{t\in\mathcal{T}}\beta_{n,k'}^t X_n + \sum_{t\in\mathcal{T}}\lambda_t^{n-1}\left(v_t^n - v_t^{n-1}\right) \\
&\quad - \sum_{t\in\mathcal{T}}\left[c_t(v_t^n) - c_t(v_t^{n-1})\right] \\
&\overset{(i)}{\geq} \eta_n + \sum_{t\in\mathcal{T}}\beta_{n,k'}^t X_n \\
&\quad + \frac{1}{\alpha}\left(D_n - D_{n-1} - \eta_n - \sum_{t\in\mathcal{T}}\beta_{n,k'}^t X_n\right) \\
&\geq \frac{1}{\alpha}\left(D_n - D_{n-1}\right),
\end{aligned}$$

where $D_n - D_{n-1} = \eta_n + \sum_{t\in\mathcal{T}}\beta_{n,k'}^t X_n + \sum_{t\in\mathcal{T}}[\hat{c}_t(\lambda_t^n) - \hat{c}_t(\lambda_t^{n-1})]$. To prove the above inequality, we only need to show that the inequality (i) holds. It is equivalent to proving that

$$\begin{aligned}
&\lambda_t^{n-1}\left(v_t^n - v_t^{n-1}\right) - \left[c_t(v_t^n) - c_t(v_t^{n-1})\right] \\
&\quad \geq \frac{1}{\alpha}\left[\hat{c}_t(\lambda_t^n) - \hat{c}_t(\lambda_t^{n-1})\right], \forall t \in \mathcal{T}, \qquad (14)
\end{aligned}$$

where the conjugate function of $c_t(v_t)$ can be derived as

$$\hat{c}_t(\lambda_t) = \begin{cases} \frac{(\lambda_t - b_t)^2}{4a_t} & \lambda_t \leq 2a_t W_t + b_t, \\ (\lambda_t - b_t)W_t - W_t^2 & \lambda_t > 2a_t W_t + b_t. \end{cases} \qquad (15)$$

Because X_n is much smaller than the capacity W_t, we can instead validate the differential version of equation (14), which is in the form of

$$\lambda_t - c_t'(v_t) \geq \frac{1}{\alpha}\hat{c}_t'(\lambda_t)f_t'(v_t), \qquad (16)$$

where $v_t = v_t^{n-1}$ and $\lambda_t = \lambda_t^{n-1}$. There are two cases.

Case I: $v_t \leq W_t/\delta$ and $\lambda_t = b_t + 2a_t\delta v_t \leq b_t + 2a_t W_t$.

$$\alpha \geq \frac{\hat{c}_t'(\lambda_t)f_t'(v_t)}{f_t(v_t) - c_t'(v_t)} = \frac{2v_t a_t \delta^2}{2a_t\delta v_t + b_t - 2a_t v_t - b_t} = \frac{\delta^2}{\delta - 1}.$$

The RHS of above equation is minimized when $\delta = 2$. In this case, we have $\alpha = 4$.

Case II: $W_t \geq v_t > W_t/\delta$ and $b_t + 2a_t W_t < \lambda_t \leq b_t + 2a_t\delta W_t$.

$$\alpha \geq \frac{\hat{c}_t'(\lambda_t)f_t'(v_t)}{f_t(v_t) - c_t'(v_t)} = \frac{W_t\xi c_t'(W_t)e^{\xi(v_t - W_t/\delta)}}{c_t'(W_t)e^{\xi(v_t - W_t/\delta)} - 2a_t v_t - b_t}.$$

We first prove an inequality

$$c_t'(W_t)e^{\xi(v_t - W_t/\delta)} \geq \frac{b_t + 2a_t W_t}{b_t + 2a_t W_t/\delta}(b_t + 2a_t v_t), \qquad (17)$$

with $\xi \geq \frac{2a_t}{b_t + 2a_t W_t/\delta}$. It is equivalent to show that

$$\frac{e^{\xi v_t}}{c_t'(v_t)} \geq \frac{b_t + 2a_t W_t}{b_t + 2a_t W_t/\delta} \frac{e^{\xi W_t/\delta}}{c_t'(W_t)} = \frac{e^{\xi W_t/\delta}}{c_t'(W_t/\delta)}. \quad (18)$$

Thus, we only need to prove that $\frac{e^{\xi v_t}}{c_t'(v_t)}$ is non-decreasing in v_t. When $\xi \geq \frac{2a_t}{b_t + 2a_t W_t/\delta}$, we can prove this by

$$\left(\frac{e^{\xi v_t}}{c_t'(v_t)}\right)' = \frac{\xi e^{\xi v_t}(b_t + 2a_t v_t) - e^{\xi v_t} 2a_t}{c_t'(v_t)^2} \geq 0.$$

On the other hand, to avoid the exceeding of the capacity, the marginal price must be no smaller than the maximum per-unit valuation from all the bids. Thus, we also have

$$f_t(W_t) = c_t'(W_t) e^{\xi(W_t - W_t/\delta)} \geq U, \quad (19)$$

which can be translated into $\xi \geq \frac{\ln(\frac{U}{c_t'(W_t)})}{W_t(1-1/\delta)}$. Thus, we choose

$$\xi = \max\left\{\frac{\ln(\frac{U}{c_t'(W_t)})}{W_t(1-1/\delta)}, \frac{2a_t}{b_t + 2a_t W_t/\delta}\right\}.$$

Based on (17), we have

$$\frac{\hat{c}_t'(\lambda_t) f_t'(v_t)}{f_t(v_t) - c_t'(v_t)} \leq \frac{\hat{c}_t'(\lambda_t) f_t'(v_t)}{f_t(v_t) - \frac{b_t + 2a_t W_t/\delta}{b_t + 2a_t W_t} f_t(v_t)} = \frac{\xi(b_t + 2a_t W_t)}{2a_t(1 - 1/\delta)}.$$

If we set $\delta = 2$, then

$$\xi = \max\left\{\frac{2\ln(\frac{U}{c_t'(W_t)})}{W_t}, \frac{2a_t}{b_t + a_t W_t}\right\}.$$

Thus,

$$\alpha = \max\left\{\frac{2(b_t + 2a_t W_t)\ln(\frac{U}{c_t'(W_t)})}{a_t W_t}, 4 - \frac{2b_t}{b_t + a_t W_t}\right\}.$$

By combining the above two cases, the competitive ratio of Algorithm 1 is

$$\alpha = \max\left\{\frac{2(b_t + 2a_t W_t)\ln(\frac{U}{c_t'(W_t)})}{a_t W_t}, 4\right\}.$$

REFERENCES

[1] A. Ipakchi and F. Albuyeh, "Grid of the future," *IEEE Power Energy Mag.*, vol. 7, no. 2, pp. 52–62, Mar./Apr. 2009.

[2] L. Gan, U. Topcu, and S. Low, "Optimal decentralized protocol for electric vehicle charging," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 940–951, May 2013.

[3] K. Clement-Nyns, E. Haesen, and J. Driesen, "The impact of charging plug-in hybrid electric vehicles on a residential distribution grid," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 371–380, Feb. 2010.

[4] W. Tang, S. Bi, and Y. Zhang, "Online coordinated charging decision algorithm for electric vehicles without future information," *IEEE Trans. Smart Grid*, vol. 5, no. 6, pp. 2810–2824, Nov. 2014.

[5] P. You, Z. Yang, Y. Zhang, S. Low, and Y. Sun, "Optimal charging schedule for a battery switching station serving electric buses," *IEEE Trans. Power Syst.*, vol. 31, no. 5, pp. 3473–3483, Sep. 2016.

[6] M. D. Galus, M. G. Vay, T. Krause, and G. Andersson, "The role of electric vehicles in smart grids," *Wiley Interdiscip. Rev.: Energy Environ.*, vol. 2, pp. 384–400, 2013.

[7] J. Cao, B. Yang, C. Chen, and X. Guan, "Optimal demand response using mechanism design in the smart grid," in *Proc. Chinese Control Conf.*, 2012, pp. 2520–2525.

[8] E. Bitar and Y. Xu, "Deadline differentiated pricing of deferrable electric loads," *IEEE Trans. Smart Grid*, vol. 8, no. 1, pp. 13–25, Jan. 2017.

[9] P. Samadi, A. H. Mohsenian-Rad, R. Schober, and V. W. S. Wong, "Advanced demand side management for the future smart grid using mechanism design," *IEEE Trans. Smart Grid*, vol. 3, no. 3, pp. 1170–1180, Sep. 2012.

[10] O. Ardakanian, C. Rosenberg, and S. Keshav, "Quantifying the benefits of extending electric vehicle charging deadlines with solar generation," in *Proc. IEEE Int. Conf. Smart Grid Commun.*, 2014, pp. 620–625.

[11] T. Liu, B. Sun, X. Tan, and D. H. K. Tsang, "Market for multi-dimensional flexibility with parametric demand response bidding," in *Proc. 49th North Amer. Power Symp.*, Sep. 2017, pp. 1–6.

[12] E. H. Gerding, V. Robu, S. Stein, D. C. Parkes, A. Rogers, and N. R. Jennings, "Online mechanism design for electric vehicle charging," in *Proc. Int. Conf. Auton. Agents Multiagent Syst.*, 2011, pp. 811–818.

[13] K. Hayakawa, E. H. Gerding, S. Stein, and T. Shiga, "Online mechanisms for charging electric vehicles in settings with varying marginal electricity costs," in *Proc. 24th Int. Joint Conf. Artif. Intell.*, 2015, pp. 2610-2616.

[14] Z. Zheng and N. B. Shroff, "Online welfare maximization for electric vehicle charging with electricity cost," in *Proc. 5th Int. Conf. Future Energy Syst.*, 2014, pp. 253–263.

[15] N. Gatsis and G. Giannakis, "Decomposition algorithms for market clearing with large-scale demand response", *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 1976–1987, Dec. 2013.

[16] A. Mualem and N. Nisan, "Truthful approximation mechanisms for restricted combinatorial auctions," *Games Econ. Behav.*, vol. 64, no. 2, pp. 612631, 2008.

[17] X. Zhang, Z. Huang, C. Wu, Z. Li, and F. C. M. Lau, "Online auctions in IaaS clouds: Welfare and profit maximization with server costs," *IEEE/ACM Trans. Netw.*, vol. 25, no. 2, pp. 1034–1047, Apr. 2017.

[18] South California Edison (SCE) Website. Accessed: Jan. 24, 2014. [Online]. Available: http://www.sce.com/005 regul info/eca/DOMSM14.DLP

[19] H. Akhavan-Hejazi, H. Mohsenian-Rad, and A. Nejat, "Developing a test data set for electric vehicle applications in smart grid research," in *Proc. IEEE Veh. Technol. Conf.*, Vancouver, BC, Canada, 2014, pp. 1–6 .

[20] N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani, *Algorithmic Game Theory*, Cambridge, U.K.: Cambridge Univ. Press, 2007.

[21] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, U.K.: Cambridge Univ. Press, 2004.

[22] C. A. Floudas, *Nonlinear and Mixed-Integer Optimization*, Oxford, U.K.: Oxford Univ. Press, 1995.

Bo Sun (S'14) received the B.E. degree in electronic and information engineering from the Harbin Institute of Technology, Harbin, China, in 2013. He is currently working toward the Ph.D. degree in electronic and computer engineering at the Hong Kong University of Science and Technology. His research interests include stochastic modeling, optimization, and pricing with applications in smart energy systems.

Xiaoqi Tan (M'18) received the B.Eng. degree in information engineering from the Xi'an Jiaotong University, Xi'an, China, in 2012. and the Ph.D. degree from the Hong Kong University of Science and Technology, Hong Kong, in 2018. From Oct. 2015 to May 2016, he was a Fellow in electrical engineering with the School of Engineering and Applied Science, Harvard University, Cambridge, MA, USA. He is currently a Postdoctoral Fellow with the Edward S. Rogers Sr. Department of Electrical & Computer Engineering, University of Toronto, Toronto, Canada. His research interests include performance analysis and optimization of communication and energy systems.

Danny H. K. Tsang (F'12) received the Ph.D. degree in electrical engineering from the University of Pennsylvania, PA, USA, in 1989. Since 1992, he has been with the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong, where he is currently a Professor. His current research interests include Internet quality of service, peer-to-peer (P2P) video streaming, cloud computing, cognitive radio networks, and smart grids. Dr. Tsang was a Guest Editor for the IEEE JOURNAL OF SELECTED AREAS IN COMMUNICATIONS special issue on Advances in P2P Streaming Systems, an Associate Editor for the *Journal of Optical Networking* (Optical Society of America), and a Guest Editor for the IEEE SYSTEMS JOURNAL. He is currently a Technical Editor for the IEEE COMMUNICATIONS MAGAZINE.