Optimal Management of Local Energy Trading in Future Smart Microgrid via Pricing

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***Abstract*—In this paper, we investigate optimal management of local energy trading in future smart micro-grid (SMG) via pricing. In SMG, energy consumers and providers, in addition to trading with utility company, can also perform local energy trading controlled by a local trading manager (LTM) for reaping benefits. We first quantify the benefits achieved by the consumers and providers from local trading and then formulate a two-layered optimization framework to investigate i) how the energy consumers and providers maximize their benefits via appropriately adjusting their local trading decisions in response to the LTM's pricing, and ii) how the LTM adjusts its price in local market to benefit the consumers and providers as much as possible while guaranteeing a required gain for itself. We propose two algorithms to solve the layered optimization problem and perform numerical experiments with practical data set to validate the proposed local trading model and the algorithms.**

I. Introduction

With a rapid growth in exploiting distributed energy resources (DERs) such as solar panels and micro-turbines, the paradigm of smart micro-grid (SMG) that facilitates local energy-transfers from DERs to neighboring energy consumers has attracted lots of interests [1], [2]. The SMG has been considered a promising approach that not only can efficiently utilize DERs but also can coordinate DERs and energy consumers to benefit macro-grid (e.g., smoothing energy demand profile and reducing demand congestion). In particular, the SMG is in line with the emerging concept of *ElectriNet* [3], where the energy users, in addition to trading with the utility companies in macro-gird, can form local energy-transfer networks for potential energy trading opportunities.

Realizing the merits of the SMG, however, necessitates a careful control of the energy trading between energy providers and consumers as well as the economic reward optimization associated with the trading. In [4], the authors investigated the energy-transfer between two energy users in an islanded micro-grid to minimize their total energy generation cost. In [5], the authors studied the distributed energy generation of multiple micro-grids for their social welfare optimization. In [6], the authors investigated the distributed charging and discharging for a group of electric vehicles in micro-grid. Market model has been commonly adopted for managing the energy consumption in SMG as well as smart grid. In [7], the authors proposed two different market models for demand response management. In [8], the authors proposed a vehicle-to-grid market model for providing frequency control service. In [9], the authors investigated the bilateral energy trading with the utility company in smart grid. Meanwhile, different layered market models have been proposed in [10], [11] for investigating the competitive consumption scheduling of energy users and the pricing of the utility companies.

Different from these previous works, we propose a novel SMG model where the energy consumers and distributed energy providers can perform local energy trading controlled by a local trading manager (LTM). The LTM adjusts its local energy price, different from the price of the utility company, to control the local trading for benefiting the energy providers, consumers as well as itself. In particular, we first quantify the benefits of the consumers and providers from local trading. Then, we formulate a two-layered optimization framework comprised of i) *a bottom-layer optimization* that models how the energy consumers and providers appropriately adjust their energy trading decisions in the local market in response to the LTM's price, and ii) *a top-layer optimization* that models how the LTM controls its price to benefit the energy consumers and providers as much as possible while guaranteeing a required gain for itself. We propose two algorithms to solve the layered optimization and validate the local trading model and the algorithms via numerical experiments.

II. System Model and Problem Formulation

A. SMG Model with Local Trading

We consider a SMG consisting of two types of energy users, i.e., a group of energy buyers (EBs) and a group of energy sellers (ESs). Let $\Omega_b = \{1, 2, ..., N_b\}$ denote the group of the EBs, in which each EB $i \in \Omega_b$ requires a fixed energy demand d_i within a certain period (e.g., one hour). Meanwhile, let $\Omega_s = \{1, 2, ..., N_s\}$ denote the group of the ESs, in which each ES $j \in \Omega_s$ has a given energy-output $\widetilde{o}_j$ to sell. In the SMG, each EB is connected to both an external utility company (EUC) and a local trading market managed by the LTM. To meet its energy demand, each EB can buy energy from both the EUC (whose *sell-out price* is p_{out}) and the LTM (whose *sell-out price* is q_{out}). Similarly, each ES is connected to both the EUC and the LTM, and it can sell its energy-output to both the EUC (whose *buy-back price* is p_{back}) and the LTM (whose *buy-back price* is q_{back}).

For each EB $i \in \Omega_b$, we use x_i to denote the energy it buys from the EUC, and use y_i to denote the energy it buys from the LTM. To meet its energy demand, each EB i requires:

$$x_i + \big(y_i - f_i(y_i)\big) \geq d_i, \forall i \in \Omega_b, \quad (1)$$

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where function $f_i(y_i)$ denotes the energy transmission loss when EB i acquires y_i from the LTM. Because of the pricing of the EUC and the LTM, EB i's energy acquisition cost is

$$C_i(x_i, y_i) = x_i p_{\text{out}} + y_i q_{\text{out}}, \forall i \in \Omega_b. \quad (2)$$

Compared to a benchmark case without the local trading, the saving of EB i when it can trade with the LTM is

$$\begin{aligned} G_i(x_i, y_i) &= C_i(d_i, 0) - C_i(x_i, y_i) \\ &= p_{\text{out}}(d_i - x_i) - q_{\text{out}} y_i, \forall i \in \Omega_b, \end{aligned} \quad (3)$$

which can be considered as the *net gain of EB i from performing the local trading*. In particular, each EB i usually requires a nonnegative net gain, i.e., requiring

$$G_i(x_i, y_i) \geq 0, \forall i \in \Omega_b. \quad (4)$$

Meanwhile, for each ES $j \in \Omega_s$, we use $\tilde{x}_j$ to denote its energy sold to the EUC and use $\tilde{y}_j$ to denote its energy sold to the LTM. Limited by its energy output $\tilde{o}_j$, ES j requires

$$\tilde{x}_j + \big(\tilde{y}_j + \tilde{f}_j(\tilde{y}_j)\big) \leq \tilde{o}_j, \forall j \in \Omega_s, \quad (5)$$

where function $\tilde{f}_j(\tilde{y}_j)$ denotes the energy transmission loss when ES j sells $\tilde{y}_j$ to the LTM. Because of the pricing of the EUC and the LTM, the total income of each ES j is

$$I_j(\tilde{x}_j, \tilde{y}_j) = p_{\text{back}} \tilde{x}_j + q_{\text{back}} \tilde{y}_j, \forall j \in \Omega_s. \quad (6)$$

Compared to a benchmark case without the local trading, the increased income of ES j when it can trade with the LTM is

$$\begin{aligned} \tilde{G}_j(\tilde{x}_j, \tilde{y}_j) &= I_j(\tilde{x}_j, \tilde{y}_j) - I_j(\tilde{d}_j, 0) \\ &= p_{\text{back}}\big(\tilde{x}_j - \tilde{d}_j\big) + q_{\text{back}} \tilde{y}_j, \forall j \in \Omega_s, \end{aligned} \quad (7)$$

which can be considered as the *net gain of ES j*. In particular, each ES j requires a non-negative net gain, i.e., requiring

$$\tilde{G}_j(\tilde{x}_j, \tilde{y}_j) \geq 0, \forall j \in \Omega_s. \quad (8)$$

In this work, we assume that the utility company's prices $(p_{\text{out}}, p_{\text{back}})$ are fixed. We focus on investigating how the LTM controls its prices $(q_{\text{out}}, q_{\text{back}})$ to motivate the local energy trading between the EBs and ESs to maximize their benefits. The details are presented below as a two-layered optimization.

B. Problem Formulation as a Two-Layered Optimization

1) ***Bottom-Layer Optimization for Energy Trading Decisions****:* In the bottom-layer, given the LTM's prices $(q_{\text{out}}, q_{\text{back}})$, the EBs and ESs determine their respective energy trading decisions by solving the following optimization problem:

$$\begin{aligned} &\text{(P1-Bottom): } H\left(q_{\text{out}}, q_{\text{back}}\right) = \\ &\max \sum_{i \in \Omega_b} U_i(G_i(x_i, y_i)) + \sum_{j \in \Omega_s} \tilde{U}_j(\tilde{G}_j(\tilde{x}_j, \tilde{y}_j)), \\ &\text{subject to: Constraints } (1), (4), (5), \text{ and } (8), \\ &\sum_{i \in \Omega_b} y_i = \sum_{j \in \Omega_s} \tilde{y}_j, \quad (9) \\ &\text{decision variables: } \{x_i, y_i\}_{i \in \Omega_b} \text{ and } \{\tilde{x}_j, \tilde{y}_j\}_{j \in \Omega_s}. \end{aligned}$$

In (P1-Bottom), (9) ensures the energy balance at the LTM, i.e., the total energy it buys from the ESs should be equal to the total energy it sells to the EBs. In the objective function, $U_i(G_i(x_i, y_i))$ measures the satisfaction of EB i for its achieved net gain $G_i(x_i, y_i)$, and $\tilde{U}_j(\tilde{G}_j(\tilde{x}_j, \tilde{y}_j))$ measures the satisfaction of ES j for its achieved net gain $\tilde{G}_j(\tilde{x}_j, \tilde{y}_j)$. The bottom problem means that the EBs and ESs adjust their energy trading decisions to maximize their total satisfaction perceived[1], under the given prices $(q_{\text{out}}, q_{\text{back}})$ of the LTM.

We use $H(q_{\text{out}}, q_{\text{back}})$ to denote the optimal value of (P1-Bottom) under the given $(q_{\text{out}}, q_{\text{back}})$ and use $\{x_i^{\text{br}}, y_i^{\text{br}}\}_{i \in \Omega_b}$ and $\{\tilde{x}_j^{\text{br}}, \tilde{y}_j^{\text{br}}\}_{j \in \Omega_s}$ to denote the corresponding optimal energy trading decisions of the EBs and ESs in response to $(q_{\text{out}}, q_{\text{back}})$. Here, "br" stands for "best response".

2) ***Top-Layer Optimization for Pricing Control****:* In the *top-layer*, based on the outcome of the bottom-layer optimization, the LTM determines its prices $(q_{\text{out}}, q_{\text{back}})$ by solving:

$$\begin{aligned} &\text{(P1-Top): } \max H\left(q_{\text{out}}, q_{\text{back}}\right) \\ &\text{subject to: } q_{\text{out}} \sum_{i \in \Omega_b} y_i^{\text{br}} - q_{\text{back}} \sum_{j \in \Omega_s} \tilde{y}_j^{\text{br}} \geq \Gamma, \quad (10) \\ &p_{\text{out}} \geq q_{\text{out}} \geq q_{\text{back}} \geq p_{\text{back}}, \quad (11) \\ &\text{decision variables: } q_{\text{out}} \text{ and } q_{\text{back}}, \end{aligned}$$

where $H(q_{\text{out}}, q_{\text{back}})$, $\{x_i^{\text{br}}, y_i^{\text{br}}\}_{i \in \Omega_b}$ and $\{\tilde{x}_j^{\text{br}}, \tilde{y}_j^{\text{br}}\}_{j \in \Omega_s}$ are the outcomes of the bottom-layer optimization (i.e., Problem (P1-Bottom)). Constraint (10) ensures that the LTM obtains its required gain, denoted by Γ, for managing the local trading. In (11), $p_{\text{out}} \geq q_{\text{out}}$ and $q_{\text{back}} \geq p_{\text{back}}$ are required such that it is beneficial for the EBs and ESs to trade with the LTM.

Remark 1: We avoid imposing constraint (10) in (P1-Bottom), which is based on the rationale that the EBs and ESs are usually unaware of the LTM's gain and only aim at optimizing their own benefits under the given prices $(q_{\text{out}}, q_{\text{back}})$.□

We emphasize that there exists a special case with $\Gamma = 0$ in (P1-Top), meaning that the LTC is *altruistic* and does not require any gain for managing the local trading at all. We will illustrate more about this special case in Section IV.B.

III. Bottom-Layer Optimization

We solve Problems (P1-Bottom) and (P1-Top) via backward induction, starting with solving (P1-Bottom) in this section and then (P1-Top) in the next one. To solve the two problems, we first make the following two assumptions.

Assumption 1: Function $U_i(z)$ (or $\tilde{U}_j(z)$) measuring the satisfaction of EB i (or ES j) is continuous, strictly increasing and concave with $U_i(0) = 0$ (or $\tilde{U}_j(0) = 0$). In particular, to account for the fairness in the achieved net gains of the EBs and ESs, we adopt $U_i(z) = \ln(1 + z)$ for each EB i and $\tilde{U}_j(z) = \ln(1 + z)$ for each ES j in this work.

The concavity of $U_i(z)$ (or $\tilde{U}_j(z)$) captures the practice that each EB i (or ES j) experiences a decreasing marginal satisfaction when its achieved net gain increases. Notice that the similar assumption also appeared in [5], [7], [10], and [11].

Assumption 2: The energy transmission loss in local trading mainly stems from the resistance effect and can be quantified by a quadratic function $f_i(y_i) = a_i(y_i)^2 + b_i y_i$ for each EB i (e.g., see [11] and [12]), where a_i and b_i are two given parameters. The same loss model also holds for each ES j.

[1]The satisfaction function has been commonly used to model the happiness of the energy users for energy services, e.g., see [5], [7], [8], [10], and [11].

We first have the following result regarding (P1-Bottom).

Lemma *1:* Given the LTM's prices $(q_{\text{out}}, q_{\text{back}})$ that meet (11), Problem (P1-Bottom) is always feasible.

Proof: There always exists a feasible case for (P1-Bottom), i.e., $y_i = \tilde{y}_j = 0, \forall i \in \Omega_b, j \in \Omega_s$ (no local trading at all). □

We next characterize the following results for (P1-Bottom).

Proposition 1: Given the LTM's prices $(q_{\text{out}}, q_{\text{back}})$ that meet (11), Problem (P1-Bottom) is a strictly convex optimization problem that accommodates a unique optimal solution.

Proof: The feasible set of (P1-Bottom) is convex. Meanwhile, the objective function can be shown as *strictly concave* by using the scalar composition rule of convex optimization theory [14]. Thus, (P1-Bottom) is a strictly convex optimization. □

Convexity of (P1-Bottom) means *zero-duality gap* between its primal and dual problems, and we thus can solve it with the Lagrangian method. Let λ denote the dual price for (9). The Lagrangian function of (P1-Bottom) can be expressed as:

$$L\left(\{x_i, y_i\}_{i\in\Omega_b}, \{\tilde{x}_j, \tilde{y}_j\}_{j\in\Omega_s}, \lambda\right) = \sum_{i\in\Omega_b} \left(U_i(G_i(x_i, y_i)) + \lambda y_i\right) + \sum_{j\in\Omega_s} \left(\tilde{U}_j(\tilde{G}_j(\tilde{x}_j, \tilde{y}_j)) - \lambda \tilde{y}_j\right). \quad (12)$$

Based on (12), given λ, the *local optimization* for EB i is

$$\underline{\text{(EB-Bottom)}}: \max_{x_i\geq 0, y_i \geq 0} U_i(G_i(x_i, y_i)) + \lambda y_i,$$
$$\text{subject to: } x_i + \big(y_i - f_i(y_i)\big) \geq d_i, \text{ and } G_i(x_i, y_i) \geq 0.$$

Meanwhile, for each ES j, its *local optimization problem* is

$$\underline{\text{(ES-Bottom)}}: \max_{\tilde{x}_j\geq 0, \tilde{y}_j \geq 0} \tilde{U}_j(\tilde{G}_j(\tilde{x}_j, \tilde{y}_j)) - \lambda \tilde{y}_j,$$
$$\text{subject to: } \tilde{x}_j + \big(\tilde{y}_j + \tilde{f}_j(\tilde{y}_j)\big) \leq \tilde{d}_j, \text{ and } \tilde{G}_j(\tilde{x}_j, \tilde{y}_j) \geq 0.$$

Followed by Proposition 1, the following result holds.

Lemma *2:* For each given λ, Problem (EB-Bottom) and (ES-Bottom) are strictly convex optimization problems that accommodate a unique optimal solution, respectively.

Proof: The proof is similar to that for Proposition 1. □

Using Proposition 1 and Lemma 2, we propose a distributed algorithm to solve (P1-Bottom). The details are presented in Algorithm (A1), which uses the bisection, i.e., from Line 2 to Line 8, to find the optimal λ for (P1-Bottom) while guaranteeing (9). For each given λ, EB i determines its energy trading decisions by solving (EB-Bottom), and ES j determines its energy trading decisions by solving (ES-Bottom). After receiving all EBs' and ESs' decisions, the LTM evaluates the gap between them in Step 6 and updates λ until convergence. The convergence of (A1) is given by Proposition 2, and its performance is shown in Fig. 2 in Section V.

Proposition 2: Given the LTM's prices $(q_{\text{out}}, q_{\text{back}})$, Algorithm (A1) is guaranteed to converge to the optimal solution of Problem (P1-Bottom) within $\log_2\left(\frac{\overline{\lambda}-\underline{\lambda}}{\epsilon}\right)$ rounds of iterations, where $\overline{\lambda}$ and $\underline{\lambda}$ are the upper and lower bounds for λ, respectively, and ϵ is the tolerance for the computational error.

Proof: Please refer to Appendix I. □

IV. Top-Layer Optimization for Pricing Control

After solving (P1-Bottom), we next solve Problem (P1-Top):

$$\text{(P1-Top)}: \max_{q_{\text{out}}, q_{\text{back}}} H\left(q_{\text{out}}, q_{\text{back}}\right), \text{subject to: constraints (10), (11).}$$

Algorithm (A1): to Solve Problem (P1-Bottom)

1: The LTM initializes $\underline{\lambda}$, $\overline{\lambda}$, ϵ, and $\delta = \epsilon + 1$.
2: **while** $|\delta| > \epsilon$ **do**
3: The LTM sets $\lambda = \frac{\overline{\lambda}+\underline{\lambda}}{2}$ and broadcasts λ to all EBs and ESs.
4: Given λ, each EB i solves (EB-Bottom) to determine (x_i, y_i) and sends y_i to the LTM.
5: Given λ, each ES j solves (ES-Bottom) to determine $(\tilde{x}_j, \tilde{y}_j)$ and sends $\tilde{y}_j$ to the LTM.
6: The LTM evaluates $\delta = \sum_{i\in\Omega_b} y_i - \sum_{j\in\Omega_s} \tilde{y}_j$.
7: If $\delta > \epsilon$, the LTM sets $\overline{\lambda} = \lambda$ and goes back to Line 2; elseif $\delta < -\epsilon$, the LTM sets its $\underline{\lambda} = \lambda$ and goes back to Line 2; else, the LTM notifies the EBs and ESs that the convergence has been reached.
8: **end while**
9: Each EB i sets $(x_i^{\text{br}}, y_i^{\text{br}}) = (x_i, y_i)$ and reports $G_i(x_i^{\text{br}}, y_i^{\text{br}})$ to the LTM. Meanwhile, each ES j sets $(\tilde{x}_j^{\text{br}}, \tilde{y}_j^{\text{br}}) = (\tilde{x}_j, \tilde{y}_j)$ and reports $\tilde{G}_j(\tilde{x}_j^{\text{br}}, \tilde{y}_j^{\text{br}})$ to the LTM.

The difficulty in solving (P1-Top) lies in that $\{x_i^{\text{br}}, y_i^{\text{br}}\}_{i\in\Omega_b}$ and $\{\tilde{x}_j^{\text{br}}, \tilde{y}_j^{\text{br}}\}_{j\in\Omega_s}$ (i.e., the outcomes of the bottom-layer) cannot be given in closed forms. Thus, the objective of (P1-Top) and (10) cannot be given analytically. Nevertheless, (P1-Top) can be solved by exhaustive search as follows (which is referred as *Algorithm (Alg-Ex)* in the rest of this work):

In the top layer: the LTM uses two-dimensional linear search over $(q_{\text{out}}, q_{\text{back}})$ subject to (11) to find the optimal prices.

In the bottom layer: for each $(q_{\text{out}}, q_{\text{back}})$ being evaluated by the top layer, the EBs and ESs perform Algorithm (A1) to determine their energy trading decisions.

A. Efficient Algorithm to Solve Problem (P1-Top)

Algorithm (Alg-Ex), however, requires a significant complexity. We thus propose an efficient algorithm to solve (P1-Top). As shown in Section V, the proposed algorithm can achieve the optimal pricing very close to Algorithm (Alg-Ex), while consuming a significantly less computational time. To design this algorithm, we first identify the following result.

Lemma *3:* Suppose that q_{back} is given. Then, the *best value* of q_{out} that maximizes the objective function of (P1-Top) is

$$\hat{q}_{\text{out}} = \arg \min_{p_{\text{out}} \geq u \geq q_{\text{back}}} \left\{ u \,\middle|\, u \sum_{i\in\Omega_b} y_i^{\text{br}} - q_{\text{back}} \sum_{j\in\Omega_s} \tilde{y}_j^{\text{br}} \geq \Gamma \right\}, \quad (13)$$

where $\{x_i^{\text{br}}, y_i^{\text{br}}\}_{i\in\Omega_b}$ and $\{\tilde{x}_j^{\text{br}}, \tilde{y}_j^{\text{br}}\}_{j\in\Omega_s}$ are the energy trading decisions in response to the LTM's prices (u, q_{back}).

Proof: Please refer to Appendix III. □

Based on Lemma 3, solving Problem (P1-Top) is equivalent to solving (here, the capital letter "E" stands for "Equivalent")

$$\text{(P1-Top-E)}: \max_{p_{\text{out}} \geq q_{\text{back}} \geq p_{\text{back}}} \hat{H}(q_{\text{back}}) = H(\hat{q}_{\text{out}}, q_{\text{back}}), \quad (14)$$
$$\text{subject to: constraint (13).}$$

Again, due to the lack of the closed expression for (13), it is still difficult to solve the above (P1-Top-E). Nevertheless, we can explore the hidden property of $\hat{H}(q_{\text{back}})$ in (14) as follows.

Property 1: $\hat{H}(q_{\text{back}})$ is unimodal for $q_{\text{back}} \in [p_{\text{back}}, p_{\text{out}}]$.

To verify the above property, Fig. 1 plots $\hat{H}(q_{\text{back}})$ by enumerating q_{back} under different parameter-settings, whose details are explained in Section V (notice that for each given q_{back}, Lemma 3 enables us to use the linear search to efficiently find $\hat{q}_{\text{out}}$). Figure 1 shows that $\hat{H}(q_{\text{back}})$ is always *unimodal*.

Remark 2: Although it is very difficult to analytically prove Property 1, Property 1 perfectly matches the influence of LTM's price q_{back} to control the local trading. Specifically, when q_{back} is small, the ESs are discouraged to trade with the LTM, which is adverse to the total benefit of the ESs and EBs. On the other hand, when q_{back} is large, the LTM requires an even greater q_{out} to meet its required gain. Hence, the EBs are discouraged to trade with the LTM, which again is adverse to the total benefit of the EBs and ESs. Thus, both small and large q_{back} are adverse, yielding the unimodal shape of $\hat{H}(q_{\text{back}})$.

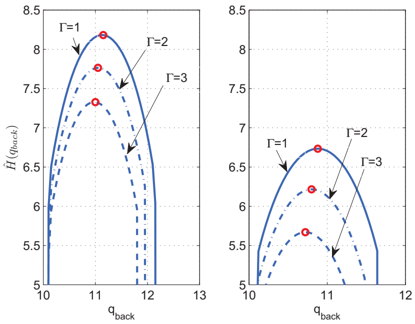

Fig. 1. Examples of $\hat{H}(q_{\text{back}})$. Left: $(p_{\text{out}}, p_{\text{back}}) = (12.5, 10)$; Right: $(p_{\text{out}}, p_{\text{back}}) = (12, 10)$. The red circle denotes the maximum of $\hat{H}(p_{\text{back}})$.

The unimodal property of $\hat{H}(q_{\text{back}})$ enables us to adopt the *Brent's method* to efficiently search for the optimal q_{back} that maximizes $\hat{H}(q_{\text{back}})$ and to solve Problem (P1-Top-E). Brent's method, which combines *the inverse parabolic interpolation and the golden section search*, is an efficient numerical method to find the maximum of a single-variable function *without requiring any first-order derivative information [13]. Brent's method is guaranteed to find the global maximum for unimodal function.* The details of Brent's method can be referred to [13], and we skip them due to the space limitation.

Using Brent's method, we propose an algorithm referred as *Algorithm (Alg-BM)*, which consists of a three-layered search listed below, to solve Problem (P1-Top-E) completely:

In the top layer: the LTM uses Brent's method to search for the optimal q_{back} within $[p_{\text{back}}, p_{\text{out}}]$.

In the middle layer: for each q_{back} being evaluated by the top layer, the LTM uses linear search to determine $\hat{q}_{\text{out}}$ within $[q_{\text{back}}, p_{\text{out}}]$ based on (13).

In the bottom layer: for each $(q_{\text{out}}, q_{\text{back}})$ being evaluated by the top and middle layers, the EBs and ESs perform Algorithm (A1) to determine their energy trading decisions.

Proposition 3: Algorithm (Alg-BM) is guaranteed to solve Problem (P1-Top) with complexity in order of $O\left([\log_2(K)]^2 K \log_2(\frac{\overline{\lambda}-\underline{\lambda}}{\epsilon})\right)$, where ϵ is the tolerance for the computational error and constant $K = \frac{p_{\text{out}}-p_{\text{back}}}{\epsilon}$. Note that $\underline{\lambda}$ and $\overline{\lambda}$ have been defined in Proposition 2 before.

Proof: First, according to Proposition 2, $\log_2(\frac{\overline{\lambda}-\underline{\lambda}}{\epsilon})$ accounts for the complexity required by Algorithm (A1) in the bottom layer with the given $(q_{\text{out}}, q_{\text{back}})$. Second, K accounts for the complexity required by the linear search for $\hat{q}_{\text{out}}$ (based on (13)) in the middle layer with the given q_{back}. Third, according to [13], Brent's method requires the complexity in order of $O\left([\log_2 K]^2\right)$ to find the optimal q_{back} in the top layer, which yields the overall complexity in Proposition 3. □

Notice that Algorithm (Alg-Ex) (described below Problem (P1-Top)) requires complexity in order of $O(K^2 \log_2(\frac{\overline{\lambda}-\underline{\lambda}}{\epsilon}))$ to solve (P1-Top). Thus, (Alg-BM) gains a significant advantage in saving computational time even for a moderate K (e.g., $K = 1000$). Figures 3 and 4 in Section V validate (Alg-BM).

B. Special Case of Altruistic LTM with $\Gamma = 0$

For the special case that the LTM is altruistic and does not require any gain for managing the local trading (i.e., $\Gamma = 0$ as described in Section II), we have the following result.

Proposition 4: If the LTM is altruistic (i.e., $\Gamma = 0$), then its optimal pricing for Problem (P1-Top) imposes $q_{\text{out}} = q_{\text{back}}$.

Proof: The result is followed by Lemma 3 directly. □

Proposition 4 matches the intuition well. To benefit the EBs and ESs as much as possible, the LTM controls its prices $(q_{\text{out}}, q_{\text{back}})$ such that it achieves the required gain exactly, i.e., (10) should be binding at the optimum of (P1-Top). Thus, when $\Gamma = 0$, the LTM sets $q_{\text{out}} = q_{\text{back}}$ such that the EBs and ESs can benefit most. Notice that in this special case, the middle layer search of Algorithm (Alg-BM) can be skipped .

V. NUMERICAL RESULTS

We perform numerical experiments by using the data set from [16]. According to [16], the residential electricity retail price in U.S. in 2014 is 12.5 cents/kWh, i.e. $p_{\text{out}} = 12.5$ cents, and the average energy consumption of each EB is approximately 1.25kWh per hour, i.e. $d_i = 1.25$kWh. To make the EBs and ESs different, $\{a_i\}_{i\in\Omega_b}$ and $\{a_j\}_{j\in\Omega_s}$ in their energy transmission loss functions are randomly chosen according to a uniform distribution within $[0.0025, 0.0075]$ (to model their different loss rates), and $b_i = b_j = 0.005, \forall i \in \Omega_b, j \in \Omega_s$.

Figure 2 shows the performance of Algorithm (A1) to solve Problem (P1-Bottom). The two subplots in the top show the results under $N_b = 10$ and $N_s = 5$ (i.e., ten EBs and five ESs), with the top-left subplot showing the convergence of λ and the top-right subplot showing the corresponding total satisfaction of all EBs and ESs. Specifically, both subplots show that (A1) quickly converges. Moreover, the top-right subplot shows that after (A1) converges, the total satisfaction of all EBs and ESs perfectly matches the optimal value of Problem (P1-Bottom) (the optimal value is obtained by the optimization solver CVX [14] and is denoted by the solid line in green), thus validating (A1). The two subplots in the bottom show the results under $N_b = N_s = 10$ and validate (A1) again.

Figures 3 and 4 show the performance of Algorithm (Alg-BM) proposed in Section VI.B to solve (P1-Top). For comparison, we also use Algorithm (Alg-Ex) to solve (P1-Top). Figure 3 plots the relative error between the optimal results (including the optimal q_{back} of the LTM and the optimal total satisfaction of all EBs and ESs) obtained by Algorithm (Alg-BM) and those obtained by Algorithm (Alg-Ex). Both subplots in Fig. 3 show that the relative errors under all tested cases are extremely small, thus validating the accuracy of

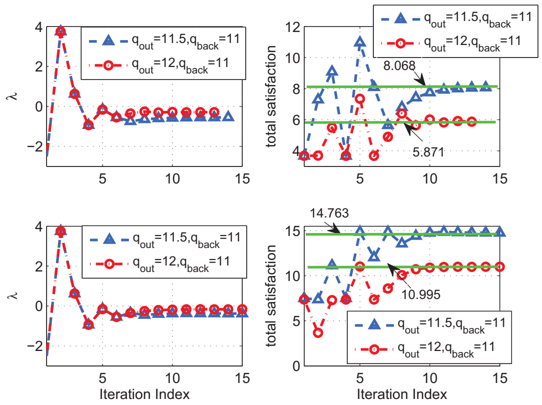

Fig. 2. Performance of Algorithm (A1) to solve (P1-Bottom). Top two subplots: $N_b = 10$, $N_s = 5$. Bottom two subplots: $N_b = N_s = 10$.

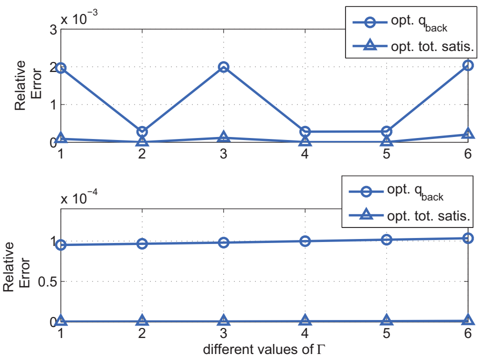

Fig. 3. Accuracy of Algorithm (Alg-BM). Top subplot: $(p_{\text{out}}, p_{\text{back}}) = (12.5, 10)$. Bottom subplot: $(p_{\text{out}}, p_{\text{back}}) = (12, 10)$. We set $N_b = N_s = 5$.

Algorithm (Alg-BM). Correspondingly, Fig. 4 further plots the computational time consumed by (Alg-BM) and (Alg-Ex). Both subplots in Fig. 4 show that Algorithm (Alg-BM) can significantly save the computational time (note that we mark the ratio of computational time saved by (Alg-BM)).

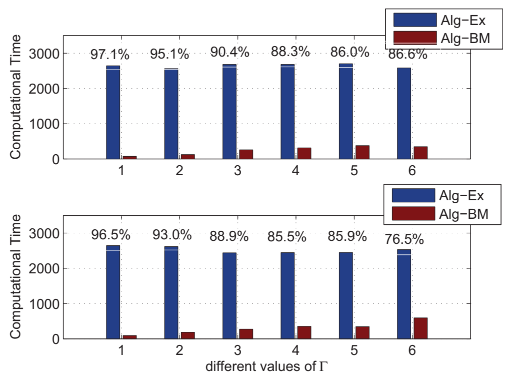

Fig. 4. Comparison of computational time. Top subplot: $(p_{\text{out}}, p_{\text{back}}) = (12.5, 10)$. Bottom subplot:$(p_{\text{out}}, p_{\text{back}}) = (12, 10)$.

Finally, Fig. 5 shows the optimal net gains achieved by all EBs and ESs from local trading. Figure 5 verifies that all EBs and ESs can positively benefit from the local trading. In particular, we evaluate the fairness level for the achieved net gains of the EBs and ESs with the metric of fairness index (FI) [15]. Figure 5 shows that the FIs are very close to one for all cases, meaning that the EBs and ESs benefit in a very fair manner, a desirable result as expected in Assumption 1.

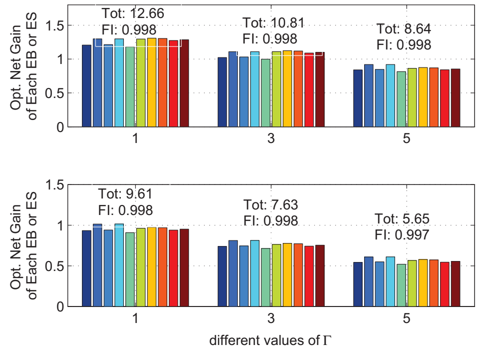

Fig. 5. Optimal net gains achieved by the EBs and ESs. Top subplot: $(p_{\text{out}}, p_{\text{back}}) = (12.5, 10)$. Bottom subplot: $(p_{\text{out}}, p_{\text{back}}) = (12, 10)$. We vary $\Gamma = 1, 3,$ and 5.

Figure 5 shows that the net gains achieved by the EBs and ESs decrease as the LTM requires to gain more. The details for this trend are further shown in Fig. 6, in which we vary Γ and plot the optimal total satisfaction of all EBs and ESs (denoted by the solid line with circles) and the sum of the achieved net gains of all EBs and ESs (denoted by the dash line with triangles). Figure 6 again verifies that the achieved net gains of the EBs and ESs are compromised as the LTM's required gain increases. Moreover, we evaluate the relative gap between the sum of all EBs' and ESs' achieved net gains (which are based on the optimal solution of the two-layered optimization) and *the potentially maximum total net gain* that they can achieve[2]. In Fig. 6, we mark the relative gaps, which are denoted by the numbers above the dash-line. Interestingly, the results show that these relative gaps are extremely small in all tested cases, meaning that the two-layered optimization enables the EBs and ESs to achieve the total net gain which is very close (with even negligible loss) to the potentially maximum total gain they could achieve, while guaranteeing that all EBs and ESs benefit in a very fair manner.

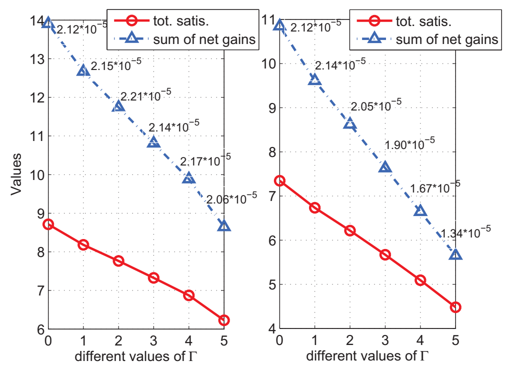

Fig. 6. Benefits of the EBs and ESs versus the LTM's required gain. Top subplot: $(p_{\text{out}}, p_{\text{back}}) = (12.5, 10)$. Bottom subplot: $(p_{\text{out}}, p_{\text{back}}) = (12, 10)$.

[2]This maximum total net gain is obtained by using $\sum_{i \in \Omega_b} G_i(x_i, y_i) + \sum_{j \in \Omega_s} \tilde{G}_j(\tilde{x}_j, \tilde{y}_j)$ as the objective function in Problem (P1-Top).

VI. Conclusion

We have investigated the optimal management of local energy trading for future SMG, in which i) the EBs and ESs perform local energy trading in response to the LTM's pricing to maximize their respective benefits, and ii) the LTM controls its price in the local trading market to benefit the EBs and ESs as much as possible while guaranteeing a required gain for itself. We formulate the whole problem as a two-layered optimization framework and propose two algorithms (i.e., (Alg-Ex) and (Alg-BM)) for achieving the optimal solution. Numerical results have been presented to validate the proposed local trading model as well as the proposed algorithms.

Appendix I: Proof of Proposition 2

To prove Proposition 2, we first present the following two lemmas (with their proofs given in Appendix II).

Lemma *4:* For each ES j, its optimal $\tilde{y}_j$ for Problem (ES-Bottom) is non-increasing in λ.

Lemma *5:* For each EB i, its optimal y_i for Problem (EB-Bottom) is nondecreasing in λ.

The above two lemmas enable us to use the bisection method to find the optimal λ ensuring that (9) is satisfied for the dual problem of (P1-Bottom). Recall that Problem (P1-Bottom) is a strictly convex optimization and thus guarantees zero duality-gap between its primal and dual problems. Thus, the bisection method can successfully find the optimal solution for Problem (P1-bottom), with the maximum iteration number no greater than $\log_2\left(\frac{\overline{\lambda}-\underline{\lambda}}{\epsilon}\right)$.

Appendix II: Proofs of Lemmas 4 and 5

We first present the proof for Lemma 4. A close look at Problem (ES-Bottom) shows that (5) should be strictly binding for achieving the optimum of (ES-Bottom) (notice that if (5) is slack, we can increase x_i further, which increases the objective function without violating any constraint). The binding of (5) enables us to substitute $\tilde{x}_j$ by $\tilde{y}_j$ and equivalently transform (ES-Bottom) into

$$\text{(ES-Bottom-E):} \quad \max_{\tilde{y}_j \geq 0 \text{ and } \tilde{y}_j + f_j(\tilde{y}_j) \leq \tilde{d}_i} \tilde{U}_j(\tilde{G}_j(\tilde{y}_j)) - \lambda \tilde{y}_j$$

$$\text{subject to: } \tilde{G}_j(\tilde{y}_j) = (q_{\text{back}} - p_{\text{back}})\tilde{y}_j - p_{\text{back}} f_j(\tilde{y}_j), \quad (15)$$

in which $\tilde{G}_j(\tilde{x}_j, \tilde{y}_j)$ in (7) is simplified into $\tilde{G}_j(\tilde{y}_j)$ in (15). By taking the first order derivative of the objective function and setting it equal to zero, we get the quadratic function below

$$-\lambda p_{\text{back}} a_j \tilde{y}_j^2 + \left(\lambda(q_{\text{back}} - p_{\text{back}}(b_j+1)) + 2p_{\text{back}} a_j\right) \tilde{y}_j + \left(\lambda - (q_{\text{back}} - p_{\text{back}}(b_j+1))\right) = 0. \quad (16)$$

A close look at (16) shows that it has two roots given by

$$r_1 = \frac{1}{2p_{\text{back}} a_j}\left(q_{\text{back}} - p_{\text{back}}(b_j+1) + 2p_{\text{back}} a_j \frac{1}{\lambda} + \sqrt{\Delta}\right),$$

$$r_2 = \frac{1}{2p_{\text{back}} a_j}\left(q_{\text{back}} - p_{\text{back}}(b_j+1) + 2p_{\text{back}} a_j \frac{1}{\lambda} - \sqrt{\Delta}\right),$$

where Δ is given by

$$\Delta = \left((q_{\text{back}} - p_{\text{back}}(b_j+1))^2 + 4p_{\text{back}} a_j\right) + 4\left(\frac{p_{\text{back}} a_j}{\lambda}\right)^2.$$

If $\lambda > 0$, then the optimal solution for (ES-Bottom-E) is

$$\tilde{y}_j = \min\{\max\{r_1, 0\}, \tilde{o}_j^e\}, \quad (17)$$

where constant $\tilde{o}_j^e = \arg\max_{z \geq 0}\{z | z + \tilde{f}_j(z) \leq \tilde{o}_j\}$ is the equivalent upper-bound for $\tilde{y}_j$. It is easy to verify that $\tilde{y}_j$ given by (17) is non-increasing when λ increases.

If $\lambda < 0$, then the optimal solution for (ES-Bottom-E) is

$$\tilde{y}_j = \min\{\max\{r_2, 0\}, \tilde{o}_j^e\}, \quad (18)$$

which again is non-increasing when λ increases.

In summary, we obtain that the optimal $\tilde{y}_j$ for Problem (ES-Bottom) is non-increasing in λ. With a slight modification of the above proof, we can also prove Lemma 5.

Appendix III: Proof of Lemma 3

It is noticed that each EB i's net gain $G_i(x_i, y_i)$ in (3) increases as q_{out} decreases. Hence, from the the view of (P1-Bottom), the smaller the q_{out}, the greater its objective function, and it is more likely that (4) is feasible. Thus, based on the definition of function $H(q_{\text{out}}, q_{\text{back}})$, we have $H(q_{\text{out}}, q_{\text{back}}) \geq H(q'_{\text{out}}, q_{\text{back}})$ if $q_{\text{out}} \leq q'_{\text{out}}$, i.e., function $H(q_{\text{out}}, q_{\text{back}})$ is decreasing in q_{out} (the result matches the intuition that a greater sell-out price q_{out} by the LTM is adverse to the net gains of the EBs and discourages the EBs to perform the local trading. As a result, the total satisfaction of all EBs and ESs decreases). Therefore, under a given q_{back}, the minimum value of q_{out} that meets (10) in (P1-Top) maximizes $H(q_{\text{out}}, q_{\text{back}})$, which yields (13) in Lemma 3.

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