Queueing Network Models for Electric Vehicle Charging Station with Battery Swapping

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***Abstract*—Electric Vehicle (EV) charging stations with battery swapping, as one promising energy supplement solution to cope with the increasing in EVs, demand a theoretical performance analysis framework. In this paper, we propose a queueing network model to serve as such a framework for battery swapping stations with a locally-charging mode. The model is a mixed queueing network with an *open queue* of EVs and a *closed queue* of batteries. Based on mild assumptions, we show the *equilibrium equations* for the queueing system, and the *steady-state distribution* is the solution of these finite equilibrium equations. In order to show the uniqueness of the solution, we prove the *ergodicity* of the system. Meanwhile, by leveraging the embedded Markov chain, we present an alternative yet much easier way to compute the steady-state distribution. Based on the steady-state distribution, various important performance indicators have been analytically determined. Simulation results demonstrate the validity of the queueing network model and reveal rich insights for the infrastructure planning of practical battery swapping stations.**

I. Introduction

Electric Vehicles (EVs) have been proposed to be the key technology to help cut down the massive greenhouse gas emissions from the transportation sector, and they are also highly expected to be the future solution to fossil fuels scarcity [1]. EVs, however, as carriers of mobile batteries with high energy storage capacity and large electric load charging requirements, will fundamentally change how electric utilities do business and will strain their existing infrastructure [1], [7]. Moreover, unlike conventional internal combustion engine vehicles, of which the gasoline tank typically can be refueled within a matter of minutes, EVs have to face the obstacle that comes from the nature of recharging: *significant amount of charging time with specific yet expensive recharging equipment* [2]. In addition, the shorter range of EVs compared to conventional vehicles necessitates more frequent recharges, which exposes EVs to another fundamental drawback, *the battery lifetime problem*.

To mitigate these negative effects, several energy supplement solutions may arise to fit different EV owners' needs [1]: i) charging stations dedicated to fleets of EVs; for instance, surprisingly, dating back to as early as 1983, Collins *et al* [3] studied the impact of EV fleets by focusing on the timing of EV recharging and its effect on utilities; ii) fast charging stations, e.g., the industrial deployment of Supercharging stations [5] and related academic work about optimal power allocation strategy among the network of fast charging stations [6]; iii) domestic or public individual charging points for slower charging, which is typically called the Level 1 or Level 2 charging [2]; and iv) last but not the least, battery swapping stations (BSSs).

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Due to the bankruptcy of the Better Place [8], an Israeli company that once hoped to revolutionize the auto industry with its innovative battery swapping business model, many people started to suspect the practicability of this concept. Nevertheless, Tesla has taken over the baton and introduced their ambitious plan to provide battery swapping services for its consumers with an impressive swapping time of around 90s [9], even faster than refueling a gas tank. The investigation of BSSs in the academic area, however, is relatively lagging behind its industrial implementation. There exist very limited recent studies on this important problem. For instance, the infrastructure planning of BSSs is formulated as a robust optimization problem in [10]. Based on this model, the author studied the potential impacts of battery standardization and technology advancements on the optimal infrastructure deployment strategy. In [11], an optimal planning framework in a distribution system for battery swapping/charging stations is proposed based on a life cycle cost analysis. The power system reliability assessment problem with EV integration using battery exchange mode is addressed in [12], and the optimization issue of using BSSs for charging EVs as well as for energy management purposes is studied in [13].

Successful development of the BSS paradigm necessitates accurate performance evaluations. Based on the aforementioned works, this paper, to the best of our knowledge, is the first to present *a theoretical performance analysis framework for BSSs with a comprehensive queueing network analysis*. The proposed model allows BSS operators to determine the relationship between the number of swapping servers, the number of charging servers and the size of parking lots. Meanwhile, performance indicators such as blocking probability, probability of immediate service, mean service time and average busy chargers will be analytically determined. We expect the work in this paper to be a benchmark for all models/algorithms that are designed to analyze the performance of BSS related systems, whether it is about infrastructure planning or operational management.

The rest of the paper is organized as follows. We introduce our queueing model of BSSs in Sec. II. In Sec. III, we obtain the steady-state distribution of this queueing network and prove the ergodicity of its embedded Markov chain, and we provide complete probability distributions of the common important performance indicators in the queueing theory context in Sec. IV. Numerical simulations and discussion are presented in Sec. V. Finally, we conclude our paper in Sec. VI.

II. Queueing Network Model

Based on the charging and delivery pattern of batteries, there exist two different ways to construct a BSS, namely the *locally-*

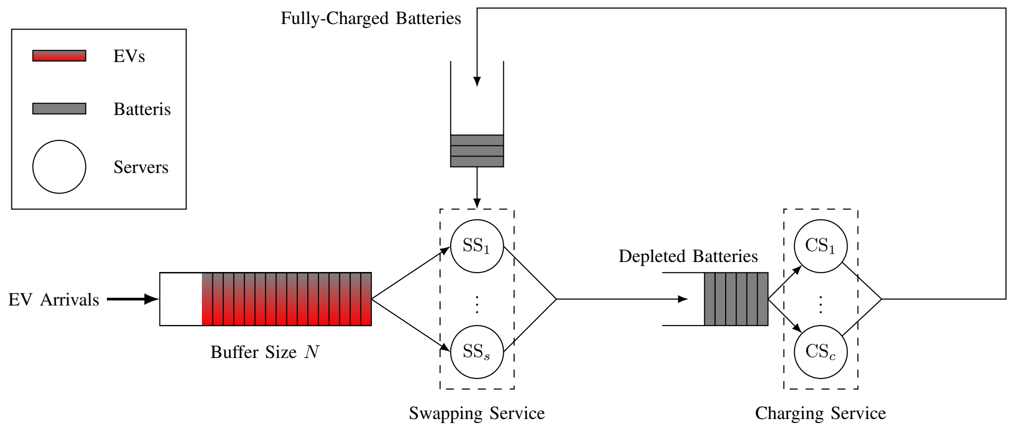

Fig. 1. The queueing network model for the EV charging station with battery swapping. The network consists of an open finite buffer queue, i.e., EV Arrivals → Swapping Service → EV Departures, and a closed queue with depleted batteries and fully-charged batteries circulating inside. We assume the two buffers in the closed queue are large enough to hold all the batteries. Therefore, there will be no deadlock and blocking in the closed queue [16]. We denote the swapping servers and charging servers as $\mathrm{SS}_i, i \in \{1, 2, \cdots, s\}$ and $\mathrm{CS}_i, i \in \{1, 2, \cdots, c\}$, respectively.

charging mode (LCM) and the *centrally-charging* mode (CCM) (both concepts will be explained later in this section). In this paper, we talk about the queueing network model of BSS with LCM, while our ongoing work is focusing on the latter. Before going into the mathematical modeling and analysis details, we first present the background of BSSs in the next subsection.

A. Problem Statement: How it Works

The battery swapping concept, as a technology or business model, is particularly analogous to cellular service providers (this analogy was first mentioned in [10], for LCM) and bottled water delivery service providers (for CCM). Cellular service providers (similar to LCM) build wireless network infrastructure and back haul infrastructure (similar to charging cables, transformers, chargers and swapping robots) and provide voice, data and video service (similar to battery refueling service). EV drivers are charged based on miles driven, exactly like cellular users, who are charged based on data usage and call time. One important feature of LCM is that batteries are charged locally within the BSS. However, for CCM, the depleted battery is swapped with a fully-charged battery and then delivered to a centralized charging station to recharge, much like a bottled water delivery service: the depleted water bottle is replaced with a new one full of water and then delivered back to a centralized facility to be refilled.

Compared to the conventional battery charging service, the battery swapping concept decouples the ownership of the battery and the EV, which are known to have different life cycles. Therefore, the battery swapping model has better potential to take advantage of future improvements in battery technology by regular battery replacement and recycling [10], [14]. Meanwhile, this ownership decoupling can significantly lower the upfront cost of buying an EV, and thus may help increase the adoption rates. Besides this, constructing BSSs with CCM in a place with massive wind or solar energy could help integrate renewable energy into the main grid more easily.

TABLE I
ABBREVIATIONS AND NOTATIONS

Symbol	Description
FCBs, FCBQ	Fully-Charged Batteries, Fully-Charged Battery Queue
DBs, DBQ	Depleted Batteries, Depleted Battery Queue
λ_{ev}	Poisson arrival rate of EVs
D_s	Deterministic swapping time, e.g., 90s
s	The total number of swapping servers
D_c	The mean charging time of a battery
c	The total number of charging servers
N	The capacity of the open queue ($N - s$ parking lots)
M	The number of batteries (customers in the closed queue)
i, n	Index to denote the number of EVs in the system
b, m	Index to denote the number of FCBs in the FCBQ
$p(i, b)$	The steady-state probability of having i EVs and b FCBs
$q(i, b)$	The steady-state probability of having i EVs waiting for service and b FCBs
α_i	The probability of having i new EV arrivals during D_s time
k	The number of batteries that have just been finished charging
$f(m, k)$	The probability of k FCBs having finished charging when there exists $M - m$ FCBs in the FCBQ

B. Queueing Network Model Specification

In this subsection, we will formally present the technical details of the proposed queueing network model. We list the abbreviations and notation symbols in Table I for reference.

1) System Overview: We consider a BSS system with LCM in Fig. 1. The system is modeled as a mixed queueing networks, which consists of an *open queue of EVs* and a *closed queue of batteries*. EVs arrive at the BSS and will be either served immediately or wait for service, and then immediately leave the system after service. During this process, each EV consumes one FCB and adds one DB to the system. Therefore, the queue of batteries being considered in this paper is closed in the sense that neither pure arrivals nor pure departures are permitted; instead each DB arrival swaps another FCB out, and this means that there exists a fixed number of batteries, which we denote by M, circulating through the network at all times.

2) Model Specifications: We now introduce some technical assumptions and specifications of the queueing network model.

- **EV Arrivals.** The arrival of EVs is assumed to follow a Poisson process with arrival rate λ_{ev}, which is considered to be measured in terms of EVs per unit time via standard vehicle traffic monitoring techniques.
- **Swapping Servers.** The total number of parallel servers is s, and each server, i.e., the battery swapping robot, is assumed to have a constant service time for each battery-swapping service. For example, Tesla's battery swapping robot can finish replacing a battery for its Model S with around 90s. We denote the service time as D_s and thus, the service rate is $\mu_s = 1/D_s$ for each server.
- **SoC Distribution.** The state-of-charges (SoCs) of the DBs are modeled on a scale between 0 and 100% (corresponding to empty and full) from a distribution with the probability density function $\phi(\mathrm{SoC})$. We denote the mean SoC by $\overline{\mathrm{SoC}}$, which is assumed to be known based on survey data.
- **Charging Servers.** Suppose the system has a total c parallel chargers with the same charging mode. Each battery, with a random SoC drawn from $\phi(\mathrm{SoC})$, is charged to full according to a specific charging curve. For example, the charging curve of the Supercharging technology of Tesla's Model S is quite linear at first and gradually slows down until full charge. In this paper, we assume the charging time is exponentially distributed with $\mu_c = 1/D_c$, which can be estimated based on $\overline{\mathrm{SoC}}$ and the given charging curve. Please see Fig. 2 for a better illustration of the relationship between $\phi(\mathrm{SoC}), \overline{\mathrm{SoC}}, D_c$ and the charging curve.

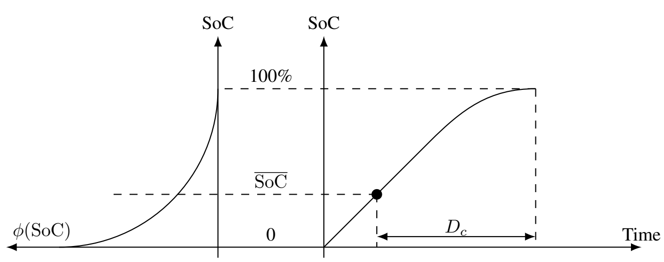

Fig. 2. Illustration of charging time distribution. Estimation of the mean service time of charging based on the SoC distribution and typical charging curve by exponential distributed processing time.

It should be made clear that while the exponential service time of the charging servers may not capture the exact charging time in practice (should be a truncated distribution), it is close to reality as well since it captures (i) the mean charging time D_c that stems from the fact that most SoCs should be around $\overline{\mathrm{SoC}}$ and (ii) the independence between different chargers and between different batteries. We also point out that all the aforementioned assumptions are motivated for mathematical tractability; however, they are widely used in the queueing theory related literature to serve as reasonable approximations and facilitate the derivation of the system structure.

In the proposed queueing network model, the queue of EVs quite resembles an $M/D/s/N$ queueing system, which is one of the classical queuing models that have been studied since the beginning of this century. However, all the EVs can only start service when the the fully-charged battery queue (FCBQ) is not empty, which makes the problem quite different from the conventional $M/D/s/N$ system. In fact, part of our proposed queueing network model is actually a token-bucket system, which has been extensively studied in the ATM network area (e.g., see [17]). However, *the strong coupling of the open queue and the closed queue makes our queueing network model very difficult to analyze purely based on what has been used in token-bucket systems*. Nevertheless, in the next section, we will derive the equilibrium equations of the queueing system and present an efficient way to obtain its steady-state distribution.

III. Steady-State Distribution and Solutions

Now we start to present the main results of this paper, namely the steady-state distribution of the queueing network. Note that the state of the network can be uniquely determined by the triple $(i, b, M-b)$, or by the tuple (i, b), where $i \in [0, N]$ denotes the number of EVs present in the system, $b \in [0, M]$ denotes the number of FCBs in the FCBQ and $M - b$ denotes the number of DBs in the depleted-battery queue (DBQ) (both include those batteries which are in the corresponding servers). Let $\mathcal{S}$ denote the state space and $S = (i, b) \in \mathcal{S}$ denote the state. Note that we will use (i, b) and $(i, b, M-b)$ interchangeably to denote the system state.

A. Exogenous and Endogenous Dynamics

We consider a continuous time model due to its nature. For any time t, we consider the time interval $(t, t + D_s]$. In the following context, α_j denotes the probability of having j new EV arrivals during this interval, i.e.,

$$\alpha_j = \frac{e^{-\lambda_{ev} D_s} (\lambda_{ev} D_s)^j}{j!} \qquad j \geq 0, \tag{1}$$

which is just the probability mass function drawn from the Poisson process with mean arrival $\lambda_{ev} D_s$. Recall that λ_{ev} is measured in unit time. Intuitively, the arrival probability (1) captures the *exogenous dynamic* of the BSS system.

We now consider the dynamic of the circulation of batteries in the closed queue, which we refer to as the *endogenous dynamic*. Let function $f(m, k)$ denote the probability of k batteries finishing charging by time $t + D_s$ and entering into the FCBQ, given that there are $M - m$ batteries in the FCBQ at time t. Followed by the i.i.d. exponential distribution of the charging time with rate $1/\mu_c = D_c$ for each server, after some basic probability calculation, function $f(m, k)$ is determined by

$$f(m,k) = \binom{\min\{c,m\}}{k} \left(1 - e^{-\frac{D_s}{D_c}}\right)^k e^{-\frac{D_s}{D_c}(\min\{c,m\}-k)},$$

where $k \in [0, \min\{c, m\}]$. Note that it is equivalent to say that the number of batteries which successfully completes charging within $(t, t + D_s]$, is a sample of size k drawn from a total population of size $\min\{c, m\}$, each of which yields success with probability $1-e^{-\frac{D_s}{D_c}}$. Note that both m and k are integers[1].

B. Steady-State Distribution

1) The Equilibrium Equations: Let $p^t(n,m)$ denote the probability of the system holding n EVs and m FCBs at time t, i.e., $S^t = (n,m)$. Due to the deterministic service time of D_s, all EVs in service at time t will have left the system and the swapped DBs will have joined the DBQ at time $t + D_s$. Consequently, all the EVs present at time $t+D_s$ either arrived during the time interval $(t, t+D_s]$ or were already waiting for service at time t; all those FCBs present at time $t+D_s$ either circulated from the DBQ during the time interval $(t, t+D_s]$ or were already waiting for EVs to service at time t. Hence, conditioned by the system state S^t present at time $t \in \mathbb{R}$,

- For $0 \le i \le N-1$ and $0 \le b \le M$, we find

$$p^{t+D_s}(i,b) = \sum_{n=0}^{N}\sum_{m=0}^{M} p^t(n,m)\alpha_{i-n+\min\{n,m,s\}} \cdot f(M-m, b-m+\min\{n,m,s\}), \tag{2}$$

  where $\min\{n,m,s\}$ denotes those EVs that have finished service at $t+D_s$ when n EVs, m FCBs and $M-m$ DBs are present in the system at t.
- For $i = N$ and $0 \le b \le M$, we have

$$p^{t+D_s}(N,b) = \sum_{n=0}^{N}\sum_{m=0}^{M} p^t(n,m)\Big(1-\sum_{\ell=0}^{N-1}\alpha_{\ell-n+\min\{n,m,s\}}\Big) \cdot f(M-m, b-m+\min\{n,m,s\}), \tag{3}$$

  which differs from (2) since all cases with new EV arrivals not less than $N-1-n+\min\{n,m,s\}$ make state $S^t = (n,m)$ transit to state $S^{t+D_s} = (N,b)$.

We shall consider the equilibrium distribution of EVs and batteries in the system. Let $p(i,b) = \lim_{t\to\infty} p^t(i,b)$ be the steady-state probability that the system is in state (i,b). Based on (2) and (3), the steady-state equations can be written as:

$$p(i,b) = \sum_{n=0}^{N}\sum_{m=0}^{M} p(n,m) g_i f(M-m, b-m+\min\{n,m,s\}) \quad \text{for } 0 \le i \le N \text{ and } 0 \le b \le M, \tag{4}$$

where g_i is the i-th entry of vector $\mathbf{g} = [g_0, g_1, \cdots, g_N]$ defined by: $g_i = \alpha_{i-n+\min\{n,m,s\}}$ for $0 \le i \le N-1$ and, $g_N = 1 - \sum_{\ell=0}^{N-1}\alpha_{\ell-n+\min\{n,m,s\}}$.

Analogously, we can also derive similar equations for $q(i,b)$ conditioned by the EV queue length, which is the steady-state distribution of the queue containing i EVs waiting and b FCBs. We omit this approach in this paper since an alternative and also much easier way to obtain $q(i,b)$ is by using $q(0,b) = \sum_{n=0}^{s} p(n,b)$ for $0 \le b \le M$, and $q(i,b) = p(i+s,b)$ for $0 < i \le N-s$ and $0 \le b \le M$.

[1]It is worth pointing out here that function $f(m,k)$ does not include the probability of one charging server finishing charging multiple batteries during interval $(t, t+D_s]$, which is definitely infeasible in practice and also small enough to be negligible mathematically.

The solution of the steady-state distribution relies on solving a finite system of $(N+1)(M+1)$ linear equations defined by (4) together with another normalization equation, $\sum_{n=0}^{N}\sum_{m=0}^{M} p(n,m) = 1$. Note that we have $(N+1)(M+1)+1$ equations but only $(N+1)(M+1)$ variables $p(0,0), \cdots, p(N,M)$. We emphasize here that in order to obtain the unique equilibrium solution, we need to exclude one of the equations in (4). Note also that the steady-state equations cannot be solved in closed form.

The linear system will have a unique steady-state solution if the corresponding embedded Markov chain is ergodic [18]. For brevity, we will refer to our embedded Markov chain as MDM-MC[2] in the following context. It is known that a given Markov chain is ergodic if it is irreducible, aperiodic and positive recurrent. However, it is non-trivial to prove the ergodicity of MDM-MC. Therefore, we give the following Theorem 1 to ensure that the finite linear system has a unique solution, regardless of which linear equation is eliminated in (4).

Theorem 1. *The embedded Markov chain MDM-MC in the battery swapping system is ergodic.*

Proof. Please see Appendix A. □

2) Computational Details: A more *compact* (in matrix form) and *convenient* (eliminates the need to solve the linear system) form of (4) can be obtained after some algebra. We would like to specify this method since it is much easier to compute in practice. In fact, the numerical solution outlined in Sec. V is also based on this method. To start with, we first define the stationary distribution by a vector as follows:

$$\mathbf{P} = [p(0,0), \cdots, p(0,M), \cdots, p(N,0), \cdots, p(N,M)].$$

We further define the state transition matrix of MDM-MC by $\mathbf{\Pi} = [\pi_{\nu,\kappa}]$, with its entries indexed by ν, κ for rows and columns, respectively (note that both ν and κ are in the range of $[1, (N+1)(M+1)]$). We also define matrix $\mathbf{\Delta} = [\delta_{j,j'}]$ with dimensionality $(M+1)\times(M+1)$ as follows:

$$\mathbf{\Delta} = \left[\begin{array}{ccc|ccc} f(0,0) & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ f(c,0) & \cdots & f(c,c) & 0 & \cdots & 0 \\ \hline f(c,0) & \cdots & f(c,c) & 0 & \cdots & 0 \\ f(c,0) & \cdots & f(c,c) & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ f(c,0) & \cdots & f(c,c) & 0 & \cdots & 0 \end{array}\right].$$

It is easy to note that $\delta_{j,j'} = f(j-1, j'-1)$, but with slight modification for some entries. The idea of defining this blocked matrix comes from a) infeasible transitions are automatically excluded by 0 entries in $\mathbf{\Delta}$ and, b) given M and c, matrix $\mathbf{\Delta}$ can be computed offline in advance (considering we have multiple candidate values of N but fixed M and c).

After the aforementioned manipulation, the construction of transition matrix $\mathbf{\Pi}$ is shown in Algorithm 1. Recall that the

[2]Following the conventional notations, M, D and M represents Poisson arrival, deterministic swapping time and exponential charging time, respectively.

stochastic matrix $\mathbf{\Pi}$ based on Algorithm 1 is ergodic according to Theorem 1. Therefore, the stationary distribution $\mathbf{P}$ exists and it is a left eigenvector with the corresponding eigenvalue of 1 of the matrix $\mathbf{\Pi}$, which implies the well-known equilibrium balance equations in matrix form $\mathbf{P\Pi} = \mathbf{P}$ augmented by the normalization equation $\sum_{n=0}^{N}\sum_{m=0}^{M} p(n,m) = 1$.

Algorithm 1: Construction of the transition matrix $\mathbf{\Pi}$

1: $\forall \nu \in [1, (N+1)(M+1)]$, define index pair (n, m) by $n = \lfloor \frac{\nu-1}{M+1} \rfloor$ and $m = (\nu - 1) \bmod (M + 1)$.
2: $\forall \kappa \in [1, (N+1)(M+1)]$, define index pair (i, b) by $i = \lfloor \frac{\kappa-1}{M+1} \rfloor$ and $b = (\kappa - 1) \bmod (M + 1)$.
3: $\pi_{\nu,\kappa} = g_{i+1}\delta_{M-m+1,b-m+\min\{n,m,s\}+1}$.

IV. Analytical Solutions of Performance Metrics

Once we obtain the steady-state probabilities, we are able to analyze the performance of the BSS system. Here we just present several of the most common performance metrics as examples due to space limitation.

A. Probability of Service Rejection/Immediate Service

Note that for any Poisson arrival system, the PASTA property holds. Therefore, the probability of blocking (PoB) $\Pr_{(B)}$ is determined by the steady state probability according to $\Pr_{(B)} = \sum_{m=0}^{M} p(N,m)$. We also define another source of "blocking", i.e., the probability of an empty FCB (PoEB), which can be calculated by $\Pr_{(E)} = \sum_{n=0}^{N} p(n,0)$. The practical rationality behind the PoEB comes from, for instance, the impatience of customers. Therefore, the probability of service rejection (PoSR), which is denoted by $\Pr_{(SR)}$, can be calculated by

$$\Pr_{(SR)} = \Pr_{(B)} + \Pr_{(E)} - p(N,0). \tag{5}$$

Note that we need to exclude the superposition $p(N, 0)$. Meanwhile, it is easy to see that if at least one swapping server is idle and at least one more FCB is available at the time of one EV arrival, that EV will be serviced immediately without any queueing. In this case, the total time spent in the system is equal to the service time and the probability of immediate service (PoIS) is

$$\Pr_{(IS)} = \sum_{n=0}^{s-1}\sum_{m=n+1}^{M} p(n,m). \tag{6}$$

B. Mean Waiting Time

The mean number of EVs in the BSS system at one time is given by $\mathscr{N} = \sum_{n=0}^{N}\sum_{m=0}^{M} np(n,m)$. Let $\mathscr{W}$ be the average waiting time. Application of Little's law yields $\mathscr{N} = \lambda_{ev}(1 - \Pr_{(B)})(\mathscr{W} + D_s)$. Therefore, the mean waiting time is

$$\mathscr{W} = \frac{\sum_{n=0}^{N}\sum_{m=0}^{M} np(n,m)}{\lambda_{ev}\big(1 - \sum_{m=0}^{M} p(N,m)\big)} - D_s. \tag{7}$$

C. Average Busy Chargers

Let β_k denote the steady-state probability of k charging servers being busy, where $0 \le k \le c$. Based on the steady-state distribution, it is easy to find that $\beta_k = \sum_{n=0}^{N} p(n, M-k)$ for $0 \le k < c$ and $\beta_c = \sum_{m=c}^{M}\sum_{n=0}^{N} p(n, M-m)$.

V. Numerical Results and Discussion

The balance equations presented in (4) have been solved numerically by using Matlab and are validated to be consistent with the result of by using Algorithm 1.

A. Simulation Setup

We have considered a BSS system with different numbers of batteries ($M = 10, 20, \cdots, 100$), different numbers of charging servers ($c = 10, 12, \cdots, 30$), different numbers of swapping servers ($s = 1, \cdots, 10$), and a buffer size/number of parking lots of $N = 2s$ (2 parking lots for each swapping robot). The arrival rate is $\lambda_{ev} = 1/30$, which means an average of 2 EVs in one minute. The swapping time D_s is assumed to be 60s, while the mean service time of the charging server is $D_c = 20$ min. (This is what the industry is managing to achieve, e.g., Tesla's next plan for its supercharging technology [5], which is somewhat beyond the current battery charging technology.) Note that all the data we have chosen are believed to be consistent with the practical system. We next present our numerical results.

B. PoB, PoEB and PoSF

The PoB, PoEB and PoSF are shown in Fig. 3. As can be seen, when fixing the number of swapping servers $s = 5$, all of these probabilities decrease smoothly with the increasing of M. When $M \le 30$, the PoB is larger than 0.1, and it decreases rapidly when M increases. Meanwhile, it is easy to observe from the figure that if we want to keep the PoSF below a threshold, say 0.1, the number of batteries M should be at least 40, and the service rejection performance does not benefit too much when we further increase the number of batteries. This is rational since when M is large enough, it is no longer the bottleneck. Likewise, when fixing the number of batteries $M = 30$, the service rejection performance with respect to c shows the same property, and it is easy to choose a suitable c with a targeted performance requirement.

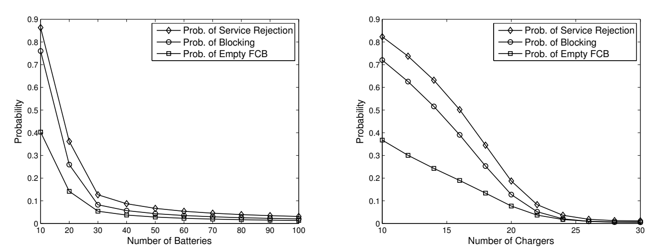

Fig. 3. Prob. of blocking, prob. of empty FCB and prob. of service rejection.

C. Mean Waiting Time

The mean waiting time and average number of EVs in the system are shown in Fig. 4. The mean waiting time is very sensitive to M since it decreases to zero sharply when M increases, and so does the average number of EVs in the system. However, both of them decrease more gently with respect to the number of chargers. From Fig. 4, it can be seen that the mean waiting time can be made small enough, e.g., less than 100s, simply by a typical setting like $M = 30$ and $c = 20$.

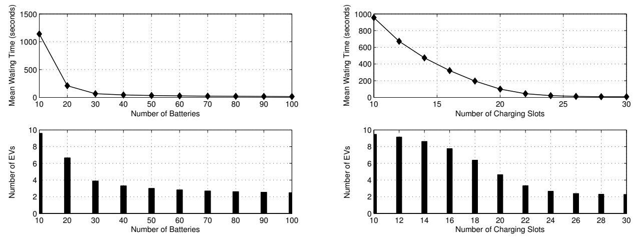

Fig. 4. Mean waiting time and average number of EVs in the system.

D. Average Busy Chargers

The average busy chargers often reflects the average charging cost in many scenarios. Therefore, we show these important results in Table II with different combinations of M values and c values. Note that when M and c are small, all the chargers are close to being busy all the time; however, when M and/or c are increased, there is some potential to have several idle chargers for some time. We emphasize this phenomenon since it is probably possible to reduce the charging cost by appropriately controlling the charging process, as well as guarantee the quality of service. In fact, considering an optimal charging policy to make a trade off between the quality of service and charging cost could be a very interesting extension of the work in this paper, which can be referred to in [15].

TABLE II
AVERAGE BUSY CHARGING SERVERS

	$c = 10$	$c = 15$	$c = 20$	$c = 25$
$M = 20$	9.9875	14.4511	$---$	$---$
$M = 30$	9.9872	14.5124	16.6437	16.3183
$M = 40$	9.9912	14.6670	17.7076	19.1481
$M = 50$	9.9934	14.7476	18.2628	20.5654
$M = 60$	9.9947	14.7972	18.6038	21.4358
$M = 70$	9.9955	14.8307	18.8344	22.0246
$M = 80$	9.9962	14.8548	19.0009	22.4495
$M = 90$	9.9967	14.8731	19.1266	22.7705
$M = 100$	9.9970	14.8874	19.2250	23.0215

VI. CONCLUSIONS

In this paper, we have proposed a queueing network model to serve as a performance analysis framework for BSSs. The problem was formulated as a mixed queueing network with an open queue of EVs and a closed queue of batteries. Based on mild assumptions, we showed the equilibrium equations for the queueing system and that the steady-state distribution is the solution of these finite linear equations. In order to show the uniqueness of the solution, we proved the ergodicity of the system, which is far from trivial. Meanwhile, based on the embedded Markov chain, we presented an alternative yet much easier way to obtain the steady-state distribution. Based on the steady-state distribution, various important performance indicators were analytically determined. Finally, we demonstrated the validity of the queueing model and showed practical insights for the design and control of future battery swapping stations.

APPENDIX A
PROOF OF THEOREM 1

We just sketch the proof due to space limitation. In fact, $\forall \kappa \in \{1, \cdots, (N+1)(M+1)\}$, and each state S_κ in MDM-MC has a self loop with a non-zero probability. According to (4), the self transition probability is given by

$$\pi_{\kappa\kappa} = g_i f(M-m, b-m+\min\{n,m,s\}). \tag{8}$$

This indicates that all the states are aperiodic, meaning the greatest common divisor of the time epochs at which one state returns to itself is 1. It is known that a finite state irreducible Markov chain is ergodic if it has an aperiodic state [18]. Therefore, it suffices to just prove MDM-MC is irreducible.

In fact, let us randomly pick two states $S_\kappa, S_{\kappa'}$, where $\kappa, \kappa' \in \mathcal{K}$. Let us denote $S_\kappa = (i,b)$ and $S_{\kappa'} = (i',b')$, with i, b, i', b' determined based on Step 2 in Algorithm 1. In order to prove $S_\kappa \leftrightarrow S_{\kappa'}$, it suffices to prove

$$(i,b) \leftrightarrow (i',b) \leftrightarrow (i',b'). \tag{9}$$

Without loss of generality, if $i' \geq i$, it is trivial to see that there is direct communication between state (i,b) and state (i',b) with a non-zero probability. We thus have established the first communicability in (9); if $b' \geq b$, the second communicability can always be satisfied for the reason that $(i',b) \leftrightarrow (i',b+1) \leftrightarrow \cdots \leftrightarrow (i',b')$. Likewise for cases that come from different combinations of $i' \geq i, i' < i, b' \geq b$ and $b' < b$.

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