Economic Analysis of Lifetime-Constrained Battery Storage under Dynamic Pricing

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***Abstract*—Battery Energy Storage System (BESS), as one type of the storage systems, serves as a particularly important role for future power grid systems. However, since both the capital cost of BESS and the potential economic value vary dramatically for large-scale systems, the total cost induced by BESS remains a major source of uncertainty for potential power system operators when the limited lifetime of BESS is taken into account. Therefore, appropriate configuration and operation of BESS are of paramount importance for its deployments in practice. In this paper, we propose a novel model for BESS that attempts to capture the fact of limited lifetime and to exploit the potential economic value. We develop a finite horizon optimization model for BESS operators with unknown stopping-time. The stopping-time is determined by the policy itself, which makes the problem technically challenging. We first propose an algorithm called Forward-iteration of relaxed-Linear Programming (FirLP), which solves the problem by iterating on every time instance and achieves the optimality. Subsequently, we observe that some time instances are not necessary to be iterated on. Thus, we propose Jump-iteration of relaxed-Linear Programming (JirLP). By utilizing a well defined jump step, we can avoid exhaustive iteration on those unnecessary time instances. We examine our model and algorithms with the real price data. The computational results further validate our model, and shows that our proposed JirLP can achieve optimality and reduce the unnecessary iterations by 50% in comparison with the FirLP.**

I. Introduction

With the rapid growing demand in electricity, concerns over carbon emission and security problems, the power grid system is foreseen to have more self-incentively participated customers enabled by bi-directional communication and widely adopted distributed energy storage devices. As a key component for power system, energy storage has an extremely broad usage including peak demands shaving, power reliability improvement, and costs reduction [1]. Based on its various usages, the number of different energy storage technologies is very large, which includes pumped hydro, compressed air energy storage, battery energy storage, super capacitors, etc.

Despite all these potential advantages of energy storage system, both the capital cost and potential economic value could vary from tens of thousands into tens of millions of dollars per year for large-scale systems. Hence, unless proper incentives and pricing policies are in place, all the aforementioned values of storage might not be fully materialized by the power system due to underinvestment [2]. Therefore, development of economic models and characterization of the associated operational policies are of paramount importance. This paper aims at providing such characterization by presenting a novel model for battery energy storage with lifetime constraint as a response to dynamic electricity price.

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In this paper, different from most of the existing literatures, we explicitly consider the limited lifetime of BESS, and highlight the effects of lifetime on the economic value of BESS. By noticing that the lifetime of BESS strongly depends on charge-behaviors and discharge-behaviors, we first propose a practical model to map the lifetime of BESS into the operational policy. Based on this model, we formulate an optimization problem to determine the optimal operational policy for the BESS to maximize its potential profit. We use the real price data to validate our model. Based on the experimental results, we conduct analysis and provide suggestions for BESS operators.

A. Related Work

The existing literatures already cover a large dimension of the energy storage problems in power system. It is known that characterizing the optimal operational policy for BESS is related to some classic inventory control problem. Nevertheless, the optimization problem for BESS is much more complicated when we take the practical properties of battery storage system into considerations, e.g., the battery's ramping constraints and the lifetime constraints. Next we discuss the related works from two main bodies as follows.

The first line of works focus on renewable energy integration. For example, Su and Gamal in [3] studied a short-time scale fast-response energy storage model in which storage and fast-ramping generation play the primary role of balancing fluctuations in demand and renewable energy power. In order to maximize a defined service lifetime/unit cost index of the energy storage system, Li *et al* in [4] proposed a dispatch strategy based on the statistics of long-term wind speed data. The strategy tries to maintain a charge-discharge-charge cycle for BESS, which is, however, somewhat unnecessary today as the development of battery technology.

The second line of works focus on economic analysis of battery energy storage, which are strongly related to our model in this paper. Harsha *et al* in [5] studied the optimal storage investment problem based on a *balancing control* mechanism. They found that for storage to be profitable under the balancing

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policy, the ratio of amortized cost of storage to the peak price of energy should be less than 1/4. The most related work to ours is [6], in which the authors analyzed the economic value of energy storage with ramp-constrained in response to stochastically varying electricity price. Specifically, the authors in [6] found that the economic value of storage capacity is a non-decreasing function of price volatility, and showed that due to finite ramping rates, the value of storage saturates quickly as the capacity increases, which is regardless of the price volatility.

B. Our Contribution

In this paper, we discuss the arbitrage value of a BESS operated under the electricity spot market. The major contributions of the paper are summarized as follows.

1) *Novel Battery Model Formulation:* We formulate the battery as a lifetime-constrained and capacity decaying queuing model. By using the *Ah-throughput Model* [7], we connect the lifetime to the detailed charge and discharge policy. Furthermore, we illustrate the detailed charge and discharge feasible region and provide a simple and yet practical model for capacity decaying process.
2) *Objective Formulation with Unknown Stopping-Time:* We formulate an optimization problem to maximize the overall profit along the lifetime of BESS, where the stopping-time depends on the control variables, *i.e.*, it is a *finite horizon optimization problem with unknown stopping-time*. To the best of our knowledge, this is the first attempt to investigate the energy storage problems using such model.
3) *Jump-iteration of relaxed-Linear Programming:* We first solve the formulated problem by brute force iteration, named as Forward-iteration of relaxed-Linear Programming (FirLP). By considering the property of the feasible policy, we further propose a low-complexity and computational-efficient algorithm called Jump-iteration of relaxed-Linear Programming (JirLP). The JirLP eliminates some time instances and directly jumps to a future time instance with a well-defined jump step, which thus avoids performing optimization over 50% of the total iterations by the FirLP.

The rest of the paper is organized as follows. In Section II, we introduce the arbitrage model of the BESS, in particular, the lifetime-constrained and capacity decaying model for battery and the reward model for operators. In Section III, we find the optimal policy and analyze the economic value of BESS. Subsequently, we use the real market electricity price data to validate our proposed model in Section IV. Based on the simulation results, we further investigate the economic value of BESS. We conclude our paper in Section V.

II. System Model and Problem Formulation

Most of the batteries work under one of the three modes (*i.e.*, charge, discharge and idle). The BESS operator decides the amount of electricity charged in (or buy from the market) and discharged out (or sell back to the market). We consider the decision is made periodically over a finite time horizon (denoted by $\mathcal{T} = \{0, 1, ..., T_m\}$, where T_m denotes the *Maximal Expected Lifetime* of the BESS, which will be specified later in this section). Let $t \in \mathcal{T}$ denote the discrete time index corresponding to the decision epoch for time interval $(t, t+1]$. We consider the following parameters for general batteries with notations specified.

- **Energy Rating** is also commonly known as the capacity of the battery with the unit of MWh. We use S to denote its initial capacity and S^t to denote the decaying process within its lifetime.
- **Power Rating** represents the maximal rate the battery can be charged or discharged at. We denote the charge and discharge profile as c^t, d^t, while using c^{max}, d^{max} for charging and discharging power rating, respectively. Power rating is also commonly known as the ramping constraints for the battery, the unit is MW or W.
- **Efficiency** consists of charge and discharge efficiencies, which are denoted by η_c and η_d, all bounded by the range of $(0, 1]$.
- **Ownership Cost** is defined as the total money spent on the BESS setup, which basically consists of initial capital cost, hardware deployment costs like wiring, housing and cooling system. We assume the cost is known to the operator, which is denoted by M.

A. Battery Model

We denote B^t as the battery level at time t, we have $\underline{B}^t \le B^t \le \overline{B}^t, \forall t$, where $\overline{B}^t, \underline{B}^t$ denote the maximal and minimal allowable level of the battery. Notice that we always have $\underline{B}^t = \gamma_1 S^t, \overline{B}^t = \gamma_2 S^t$, where γ_1 and γ_2 are coefficients which are determined by the depth of discharge. Based on the action $\{c^t, d^t\}$, the battery level B^t evolves according to

$$B^{t+1} = B^t + \eta_c c^t - \frac{1}{\eta_d} d^t \tag{1}$$

Considering the power rating constraints and the limited capacity, we have

$$0 \le c^t \le \min\{\overline{B}^t - B^t, c^{max}\} \tag{2}$$

$$0 \le d^t \le \min\{B^t - \underline{B}^t, d^{max}\} \tag{3}$$

There exists a large number of works discussing the lifetime modeling for various kinds of batteries. Among all these models, the *Ah-throughput Model* assumes that there exists a *fixed amount of energy* that can be cycled through a battery before it needs to be replaced, and more importantly, this fixed amount is independent of the depth of the individual cycles or any other parameters specific to the way the energy is drawn in or out of the battery [7]. With this, we assume that the *rating throughput* of our BESS is denoted by Θ_m. The throughput already used up to time t is then defined as

$$\Theta^t = \begin{cases} 0 & t = 0 \\ \sum_{\tau=0}^{t-1}(\eta_c c^\tau + \frac{d^\tau}{\eta_d}) & t \in \mathcal{T} \backslash \{0\} \end{cases} \tag{4}$$

Therefore, Θ^t should be bounded by $[0, \Theta_m]$. Notice that by using the *Ah-throughput model*, the lifetime of BESS can be interpreted as the longest time a BESS can last before the total

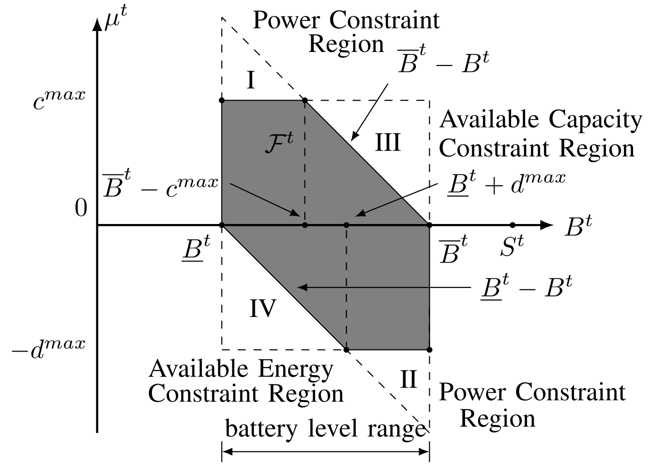


Fig. 1. Feasible action region for battery. In this figure, we define $\mu^t = \eta_c c^t - d^t/\eta_d$ as the net energy flow through the BESS, *i.e.*, $c^t = \frac{\max(\mu^t,0)}{\eta_c}$, $d^t = \eta_d \max(-\mu^t, 0)$. The efficiency $\eta_c = \eta_d = 1$ for simplicity. Note that $c^t \cdot d^t = 0$, means at most only one of them has positive value, which means the battery can only be in one of the working modes.

used throughput up to time t is less than or equal to the rating total throughput Θ_m, which is given by

$$\sup \{t | \underbrace{\sum_{\tau=0}^{t-1} (\eta_c c^\tau + \frac{d^\tau}{\eta_d})}_{\Theta^t} \leq \Theta_m\} \tag{5}$$

Based on the above Eq.(5), we transfer the lifetime into charge and discharge policy. Therefore, the lifetime of BESS is directly related to how the battery is used. As we have indicated in Section I, in practice, the capacity of BESS will decay during its lifetime. An easy observation is that, at the beginning of the operation, $S^0 = S$, which is the initial level of capacity, while at the end of lifetime, we have $S^t = \rho S$, where t is obtained from (5), ρ is a constant, and a typical value for ρ is 80%[1]. Before modeling the capacity decaying process, we first make the following assumption.

Assumption 1: The capacity will degrade when the battery is working on charge and discharge modes while it will remain unchanged in idle mode.

Based on Assumption 1 and the aforementioned boundary condition, S^t can be formulated as (suppose that the capacity is linear in Θ^t)

$$S^t = S \cdot (1 - \frac{1-\rho}{\Theta_m} \cdot \Theta^t) \tag{6}$$

Basically, (6) captures the fact that the battery's lifetime decreases quickly if it is charged/discharged frequently, and its lifetime remains unchanged if it is idle. Here we ignore the environmental effects and the battery leakage.

We summarize our battery model in Fig. 1. The electricity level B^t is always bounded within the region $[\underline{B}^t, \overline{B}^t]$. The gray area is the feasible action region, which is denoted as $\mathcal{F}^t$, and the other four regions are the two Power Constraint Regions (*i.e.*, I,II), the Available Capacity Constraint Region (*i.e.*, III) and the Available Energy Constraint Region (*i.e.*, IV). Since the capacity S^t will decrease during the operation, a quick observation is that the grey region will gradually shift to the left. Another important observation is that the feasible action region is a convex polyhedron, which thus facilitates our following solution methodology.

B. Objective for BESS Operators

To capture the economic value as well as the limited lifetime property of BESS for operators, we consider a finite-time horizon. Recall that the terminal time of the battery storage depends on the operation policy, *i.e.*, the charge and discharge profile of each time slot along the lifetime. Therefore, the time horizon in our problem is a function of c^t and d^t.

Recall that we use the constant T_m to denote the *Maximal Expected Lifetime* of the BESS. Alternatively, T_m means the *exit time*[2] of the portfolio (*i.e.*, the BESS) for the investor (*i.e.*, the BESS operator). Since the lifetime of battery defined in (5) can be either shorter or longer than T_m, the time horizon in our profit-maximization problem can be formulated as

$$\mathcal{T}(\mathbf{c}, \mathbf{d}) = \min \left\{ T_m, \sup \{t | \Theta^t \leq \Theta_m\} \right\} \tag{7}$$

where $\mathbf{c} = (c^\tau)_{\tau=0}^t$ and $\mathbf{d} = (d^\tau)_{\tau=0}^t$ represent the charge/discharge profile sequence up to time t. The above Eq.(7) means that the stopping epoch of the time horizon should be the minimal between the pre-estimated epoch T_m and the real terminal epoch of the BESS.

We model the reward function as the total money we make by selling electricity minus the total money of buying electricity from the market plus the holding cost. Thus, the reward $R(c^t, d^t, \hat{p}^t)$ in period t is a function of the triple $(c^t, d^t, \hat{p}^t)$, and it can be formulated as

$$\begin{aligned} R(c^t, d^t, \hat{p}^t) &= (\hat{p}^t - \frac{\alpha}{\eta_d}) d^t - (\hat{p}^t + \alpha \eta_c) c^t - h^t B^t \\ &= \begin{cases} -(\hat{p}^t + \alpha\eta_c)c^t - h^t B^t & c^t \in \mathbb{R}_+, d^t = 0 \\ -h^t B^t & c^t = 0, d^t = 0 \\ (\hat{p}^t - \frac{\alpha}{\eta_d}) d^t - h^t B^t & d^t \in \mathbb{R}_+, c^t = 0 \end{cases} \end{aligned}$$

Remark 1: By using the *Ah-throughput model*, the cost of *per unit electricity use* of the battery is denoted by $\alpha = \frac{M}{\Theta_m}$, which is a proportional coefficient mapping the charge and discharge profile into the monetary cost. h^t is the holding cost factor for battery which is assumed to be known to the operator. h^t depends on the working condition like weather, temperature, etc. $\hat{p}^t$ is the predicted dynamic electricity price at time t. Since in this work we only focus on the pre-assessment of the economic value of BESS, we do not explicitly consider any specific prediction model. We assume that the price is obtained by the state-of-the-art prediction method and it remains deterministic in our model[3]. Actually, for the

[1] ρ is defined as the threshold for capacity decaying below which the operator is obligated to replace the battery. For example, if a battery that has been operating for years is only able to supply 70% of its nominal capacity, the battery is considered dead [7].

[2] Exit time of a portfolio means the time when the portfolio will be liquidated. Based on the BESS and market analysis, the operator of the BESS only cares about how much profit it can achieve during the period of T_m hours. Detailed information can be referred to [8].

[3] We assume that the BESS operator in this paper is a small one compared to the whole market. Thus, the decision made by this operator will not affect the pricing mechanism in the whole market.

deployment of BESS, operators are always interested in the potential reward in the 3-5 years [1], which implies that, it will be impractical to use the existing state-of-the-art electricity price prediction models to make forecasts in such a long time in advance.

Remark 2: Here, we explicitly formulated the reward as a piecewise function in three parts, which corresponds to the three working modes of the BESS. Note that, charging and discharging at the same time will not be optimal for sure, since we impose *per unit electricity use* cost for the BESS, *i.e.*, the value of α.

Therefore, the problem can be formulated as a deterministic optimization problem over a finite horizon but with an unknown stopping-time as follows

$$\begin{aligned} \underset{\{d^t, c^t\}}{\text{maximize}} \quad & \left\{ \sum_{t=0}^{\mathcal{T}(\mathbf{c},\mathbf{d})-1} \Big[R(c^t, d^t, \hat{p}^t) \Big] \right\} \\ \text{subject to} \quad & (1) - (7) \end{aligned} \tag{8}$$

We maximize the reward over the time horizon which either stops at epoch T_m (i.e., the case that the BESS lasts longer than what we expect) or at the end of BESS's lifetime (i.e., the policy ends the life of battery before T_m). The objective function is piecewise linear, and all constraints are linear in c^t and d^t. However, the stopping-time $\mathcal{T}(\mathbf{c},\mathbf{d})$, which determines the dimension of the LP, is coupling with the control variables, and thus makes the problem hard to solve. This difficulty motivates us to design efficient algorithms to solve this problem.

III. Optimal Policy and Economic Value of BESS

An intuitive way to use battery is to charge it when the price is low and discharge it when the price is high. However, in general, it is impossible to quantify such a threshold which separates the "high" price and "low" price region, because the threshold may depend on the current level of battery and the throughput already used up to the current stage. For the same problem with given stopping-time, *i.e.*, when $\mathcal{T}(\mathbf{c},\mathbf{d})$ reduces to a constant, we can solve it directly by linear programming. In fact, this case corresponds to that when the real battery life is longer than T_m. In other words, the battery is good enough such that it never stops working before T_m. In this paper, we consider a more general case that the battery may end its life before T_m or after T_m. We next give the concept of *optimal policy* and *economic value* of BESS under this general case.

Definition 1: For a BESS operator, the *optimal policy* is a vector of tuple $(\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)} = (c^{t,*}, d^{t,*})_{t=0}^{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)} \in \mathcal{F}^0 \times \mathcal{F}^1 \times \cdots \times \mathcal{F}^{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)}$ such that, $\forall (\mathbf{c},\mathbf{d})_{\mathcal{T}(\mathbf{c},\mathbf{d})} \neq (\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)}$, we have

$$V\Big((\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)} \Big) \geq V\Big((\mathbf{c},\mathbf{d})_{\mathcal{T}(\mathbf{c},\mathbf{d})} \Big) \tag{9}$$

Where the left-hand-side of (9) represents the *economic value* of BESS, which is defined as the maximal potential profit as follows

$$V\Big((\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)} \Big) = \sum_{t=0}^{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)-1} R(c^{t,*}, d^{t,*}, \hat{p}^t) \tag{10}$$

According to Definition 1, we know that the *optimal policy* should achieve two goals *simultaneously*: 1) the battery will end its life at $\mathcal{T}(\mathbf{c}^*, \mathbf{d}^*)$, which is, either terminating at epoch T_m or ending before T_m when $\sup\{t|\Theta^t \leq \Theta_m\} < T_m$; 2) the potential profit is maximized.

A. Forward-iteration of relaxed-Linear Programming

We first propose an algorithm to solve problem (8). The basic idea of this algorithm is to relax constraint (7) about the stopping-time $\mathcal{T}(\mathbf{c},\mathbf{d})$ and to change it to a conventional finite horizon problem with a given stopping-time.

Specifically, suppose that we relax constraint (7) and change problem (8) to the conventional finite horizon problem with time index $t \in \{0, 1, \cdots, T\}$. Meanwhile, to guarantee the feasibility, we also add another constraint:

$$\sum_{t=0}^{T-1} (\eta_c c^t + \frac{d^t}{\eta_d}) \leq \Theta_m \tag{11}$$

After these operations, the objective function with constraints (1)-(4), (6) and (11) forms a standard LP as follows

$$\begin{aligned} \underset{\{d^t, c^t\}}{\text{maximize}} \quad & \sum_{t=0}^{T-1} \Big[R(c^t, d^t, \hat{p}^t) \Big] \\ \text{subject to} \quad & (1), (2), (3), (4), (6), (11) \end{aligned} \tag{12}$$

Problem (12) is a relaxed version of problem (8), and we name it as r-LP for simplicity. Based on r-LP, we give the following Theorem 1, which characterizes the optimal policy and economic value of BESS for problem (8).

Theorem 1: $\forall T = [1, T_m]$, we obtain the optimal policy $(\mathbf{c}^*, \mathbf{d}^*)_T$ and economic value $V\big((\mathbf{c}^*, \mathbf{d}^*)_T\big)$ for r-LP by iterating each T. We denote $\mathcal{T}_\Theta = [1, T_m] \bigcap \{T|\Theta^T = \Theta_m\}$. Then the optimal policy for problem (8) is obtained by the following conditions:

$$\begin{cases} V\big((\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)}\big) = \max\limits_{\{T \in \mathcal{T}_\Theta\}} \big\{ V\big((\mathbf{c}^*, \mathbf{d}^*)_T\big) \big\} \\ (\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)} = \underset{\{(\mathbf{c}^*,\mathbf{d}^*)_T | T \in \mathcal{T}_\Theta\}}{\arg\max} \big\{ V\big((\mathbf{c}^*, \mathbf{d}^*)_T\big) \big\} \end{cases} \tag{13}$$

The rationale behind Theorem 1 is to enumerate all the possible stopping-epoch from 1 to T_m. Then, the optimal policy must be the one such that it satisfies the condition (13). Theorem 1 is based on the Forward-iteration of relaxed-Linear Programming (FirLP). We omit the proof in this paper.

Remark 3: FirLP is intuitive and straightforward. However, the computational complexity is high when the time horizon is long and thus hinders its usage in practice (e.g., for 3-5 years of time horizon). Hence, we further propose a low-complexity algorithm to solve problem (8) in the next section.

B. Jump-iteration of relaxed-Linear Programming

Based on Definition 1, if we have a feasible policy which can operate BESS over a longer time horizon, it is unnecessary to check what the economic value of BESS is over a shorter one. The reason is that the operator will always have incentive to use it if the BESS can physically last to future time. Based on this observation, we can simply skip some of the time instances and directly jump to a far away time instance by a certain jump step. Therefore we propose another low-complexity and computationally efficient algorithm to solve problem (8) with guaranteed optimality, which is named Jump-iteration of relaxed-Linear Programming (JirLP).

For JirLP, we use the same input information as FirLP. However, instead of iterating over each stage before T_m, we check constraint (7) to determine the next time instance to be iterated on. More specifically, if $\Theta^{T-1} = \sum_{t=0}^{T-1}(\eta_c c^t + \frac{\eta_d}{d^t}) = \Theta_m$, we simply jump to the next stage, which is the same as the FirLP. Otherwise, we calculate the gap between Θ^{T-1} and Θ_m, which is denoted by Δ_Θ, and then obtain the *safe jump step* Δ_T and directly jump to stage $T + \Delta_T$ to be iterated on[4]. The pseudo code for JirLP is shown below as Algorithm 1.

Algorithm 1: JirLP

Data: $\hat{p}^t, B^0, S, T_m, \Theta_m, \rho, h^t, \gamma_1, \gamma_2, \eta_c, \eta_d, c^{max}, d^{max}$
Result: $(\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)}, V\big((\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)}\big), \mathcal{T}(\mathbf{c}^*, \mathbf{d}^*)$
begin
- $T \longleftarrow 1$;
- $\mathcal{T}_\Theta \longleftarrow \emptyset$;
- **while** $T \le T_m$ **do**
  - obtain the optimal policy $(\mathbf{c}^*, \mathbf{d}^*)_T$ and economic value $V\big((\mathbf{c}^*, \mathbf{d}^*)_T\big)$ by solving the r-LP Problem;
  - **if** $\sum_{t=0}^{T-1}(\eta_c c^t + \frac{\eta_d}{d^t}) = \Theta_m$ **then**
    - $T \longleftarrow T + 1$;
    - $\mathcal{T}_\Theta = \mathcal{T}_\Theta \bigcup \{T\}$;
  - **else**
    - $\Delta_\Theta \longleftarrow \Theta_m - \sum_{t=0}^{T-1}(\eta_c c^t + \frac{\eta_d}{d^t})$;
    - $\Delta_T \longleftarrow \lceil \frac{\Delta_\Theta}{\max\{\eta_c c^{max}, d^{max}/\eta_d\}} \rceil$;
    - $T \longleftarrow \min\{T_m, T + \Delta_T\}$;
  - **end**
- **end**
- $V\big((\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)}\big) = \max_{\{T \in \mathcal{T}_\Theta\}} \big\{V\big((\mathbf{c}^*, \mathbf{d}^*)_T\big)\big\}$;
- $(\mathbf{c}^*, \mathbf{d}^*)_{\mathcal{T}(\mathbf{c}^*,\mathbf{d}^*)} = \arg\max_{\{(\mathbf{c}^*,\mathbf{d}^*)_T | T \in \mathcal{T}_\Theta\}} \big\{V\big((\mathbf{c}^*, \mathbf{d}^*)_T\big)\big\}$;
- $\mathcal{T}(\mathbf{c}^*, \mathbf{d}^*) = \arg\max_{\{T | T \in \mathcal{T}_\Theta\}} \big\{V\big((\mathbf{c}^*, \mathbf{d}^*)_T\big)\big\}$;

end

The principle behind the JirLP is similar to the idea of *slow start* in the Transportation Control Protocol (TCP) for computer communication networks. When the throughput is far from being used up (analogy to that the transmission condition is good), we jump with a large step (analogy to the exponential increase of TCP congestion window). On the other hand, when the throughput is already used up (analogy to that the transmission condition is bad, *i.e.*, congestion or large packet loss), we jump with a step equal to 1 (analogy to TCP's congestion avoidance phase). The JirLP can achieve optimal policy since it will never miss any time instances when the throughput is used up, i.e., for those $T \in \mathcal{T}_\Theta$.

IV. Computational Experiment

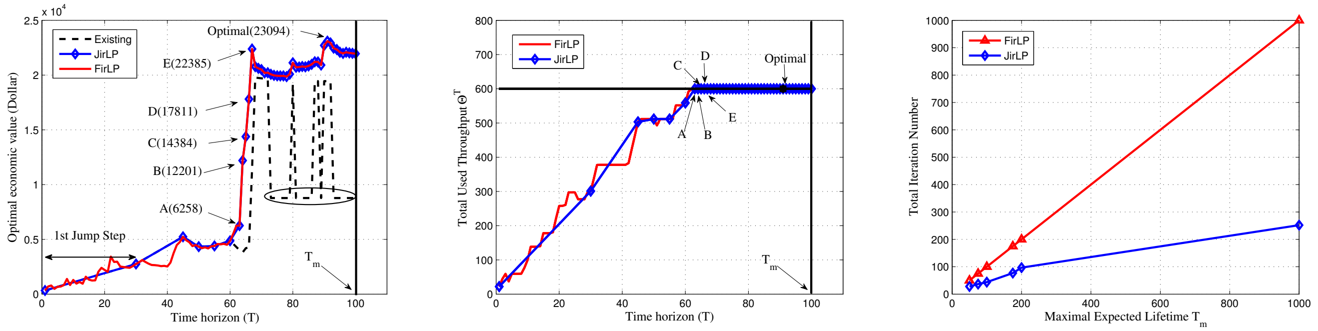
In this section, we employ numerical computations to examine our battery model and validate our assumptions. The setup of the dynamic electricity price is as follows. As stated in Section II-B, we focus on the pre-assessment of economic value of BESS, thus we do not explicitly consider the detailed model to predict data $\hat{p}^t$. We apply the model to the real-time wholesale market data taken from NYISO[9]. NYSIO reports the real time price for every five minutes from 1999 until present. We take the actual data from NYISO for the whole year of 2012, and then average over one hour to get the hourly price. For the BESS, we assume $\alpha = 10$, and the capacity is assumed initially to be $S = 100$MWh with the power rating $c^{max} = d^{max} = 20$MWh. we assume the rating throughput $\Theta_m = 600$MWh. Due to the space limitation, here we choose the price between $[0, 100]$ and simulate the time horizon with $T_m = 100$ hour[5].

We show the potential economic value of BESS in Fig. 2(a). By using the FirLP, we are iterating over each possible time instance, which is shown as the red curve in Fig. 2(a), while as we expect, the JirLP will directly jump over those small time instances since they are impossible to be the stopping-time. Both algorithms will iterate on every time instance when the rating throughput is used up, since it is possible for each time instance to have the optimal profit. We also illustrate the economic value achieved by the existing scheme when lifetime issue is not taken into consideration. As shown by the dashed curve in Fig. 2(a), when the total used throughput is not greater than the rating throughput, both the economic value achieved by the existing scheme and our approach have the same performance, which is shown by the overlap of the three curves when $T \le 60$ in Fig. 2(a). However, when the total used throughput is over the rating throughput, the economic value of the existing scheme suffers from sharp decrease, which is shown by the circle in Fig. 2(a). The reason is that, the "optimal" policy obtained by neglecting the lifetime constraint cannot be completely implemented in reality (truncation of the scheduled policy). In Fig. 2(b), we show that both the total used throughput obtained by the FirLP and the JirLP will eventually be bounded by the rating throughput Θ_m. We also compare the efficiency under different time horizon of T_m. As shown in Fig. 2(c), by using the JirLP, we can avoid iterating on over 50% of the total iteration by the FirLP. One interesting observation from Fig. 2(a) is that, even though points A, B, C, D and E represent operating the BESS with the same throughput (all bounded by Θ_m), which correspond to the five points in Fig. 2(b). However, the total economic value achieved is very much different. This tells us that, even under the same price situation and exactly the same BESS, different policies will result in totally different economic value.

Fig. 3 shows that the trajectory of the BESS operated under the optimal policy when $T_m = 100$, and the optimal policy terminates the lifetime of BESS at time $t = 91$, which can be seen from Fig. 2(a) and Fig. 2(b). As shown in Fig. 3(a), the capacity will decay when the BESS is charging or discharging and remain unchanged in idle. The BESS eventually decays to 80% of the initial capacity since we assume $\rho = 0.8$. Fig. 3(b) shows the charge and discharge profile in each stage over the entire time horizon. We can observe that due to the per unit usage cost of BESS, the BESS will keep idle for most of the

[4]In Algorithm 1, in the calculation of Δ_T, we use the symbol $\lceil x \rceil$ to denote the smallest integer that is larger than or equal to x.

[5]This simulation could be readily extended to thousands of time horizons.



(a) Economic Value of BESS (b) Throughput Sample Path (c) Efficiency Comparison

Fig. 2. Economic value and throughput sample path for the BESS. Comparison of computational efficiency between the FirLP and the JirLP.

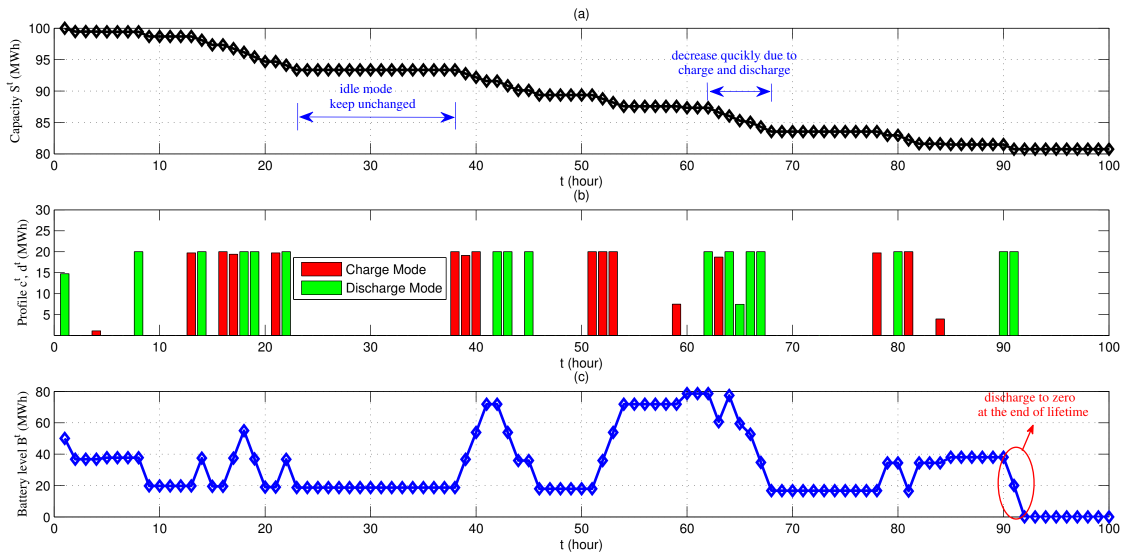


Fig. 3. Trajectory for the BESS operated on the optimal policy; (a) capacity decaying process;(b) charge and discharge profile each stage;(c) battery level.

stages to avoid the loss from inappropriate charges/discharges. Meanwhile, based on the current price and the current level of BESS, the BESS will switch its working mode between idle, discharge and charge, as depicted both in Fig. 3(b) and Fig. 3(c). The level of BESS will eventually shrink to the minimal when it reaches the lifetime of BESS, as shown in the red circle in Fig. 3(c).

V. Conclusion

In this paper, we propose a novel model to exploit the potential economic value of the BESS with an explicit lifetime constraint. We develop a finite-time horizon optimization model for the BESS operator to maximize its profit under an unknown stopping-time. We first propose the FirLP, which solves the problem by iterating on every time instance. We further observe that some of the time instance are not necessary to be iterated on. Thus, we propose the low-complexity algorithm JirLP by eliminating some of the unnecessary time instances during the search. The results show that both the FirLP and the JirLP achieve the optimal policy, while the JirLP can avoid optimizing over 50% of the total iterations by the FirLP.

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