Mobile Netw Appl
DOI 10.1007/s11036-015-0645-9

Optimal Downlink Scheduling for Heterogeneous Traffic Types in LTE-A Based on MDP and Chance-Constrained Approaches

Samira Niafar[1] · Xiaoqi Tan[1] · Danny H.K. Tsang[1]

Abstract The current mobile broadband market experiences major growth in data demand and average revenue loss. To remain profitable from the perspective of a service provider (SP), one needs to maximize revenue as much as possible by making subscribers satisfied within the limited budget. On the other hand, traffic demands are moving toward supporting the wide range of heterogeneous applications with different quality of service (QoS) requirements. In this paper, we consider two related *packet scheduling* problems, i.e., long-term and short-term approaches in the 4th generation partnership project (3GPP) long term evolution-advanced (LTE-A) system. In the long-term approach, the long-term average revenue of SP subject to the long-term QoS constraints for heterogeneous traffic demands is optimized. The problem is first formulated as a constrained Markov decision process (CMDP) problem, of which the optimal control policy is achieved by utilizing the channel and queue information simultaneously. Subsequently, in the short-term approach, we consider the short-term revenue optimization problem which stochastically guarantees the short-term QoS for heterogeneous traffic demands through a set of chance constraints. To make the proposed chance-constrained programming problem computationally tractable, we use the *Bernstein approximation* technique to analytically approximate the chance constraint as a convex conservative constraint. Finally, the proposed packet scheduling schemes and solution methods are validated via numerical simulations.

✉ Samira Niafar
sniafar@ust.hk

1 The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong

Keywords Chance-constrained · Constrained Markov decision process · Bernstein approximation · LTE-A · Heterogeneous delay requirements

1 Introduction

As mobile broadband traffic demand shifts from voice-dominated to data-dominated traffic, the SP's revenue is not keeping pace with the dramatic increase in traffic volume [8]. In order to remain profitable, SPs are looking at ways to reduce their costs and improve their revenues. On the other hand, subscribers require SPs to ensure their QoS for the wide range of heterogeneous services. To provoke such a scheme to track the revenue rather than the demand, while fulfilling the stringent QoS guarantees, one requires an effective *resource scheduling* scheme. The 3GPP LTE-A, as the fourth generation of cellular network mobile communication standard, promotes a flexible resource scheduling by allowing SP's desired algorithms to be developed. However, all the key parameters required to design a resource scheduler such as all signalling and users' QoS requirements are specified in details in the 3GPP LTE-A standard [14].

In LTE-A, the base station (eNodeB) schedules units of time-frequency resources known as *resource blocks* among LTE users. It is trivial to show that SP can maximize its revenue by allocating the resource blocks to the users which make the best profit based on channel condition. However, this resource allocation approach may suffer from the violations of the 3GPP LTE-A scheduling constraints and QoS requirements as described in the following: First, although orthogonal frequency division multiple access (OFDMA) as the downlink radio access technology of the LTE-A system allows multiple resource blocks with different data rates to be assigned to a single user, 3GPP standard does not support

Published online: 17 September 2015

multiple simultaneous data rates for a single user in order to avoid excessive signaling overhead. Thus, to have a 3GPP standard-compliant resource scheduler we select a common modulation and coding scheme (MCS) over all resource blocks assigned to a user in our scheduling policy (refer to section 10.2 in [1]). This constraint is previously considered in multiple related works such as [9] and [18], whereas the formulated scheduling problems proved as NP-hard in these works. Authors in [11] achieved the optimal solution for the same scheduling scheme by proving the total-unimodularity of the reformulated problem and solved it as an standard linear programming problem. Second, the control policies which are only adaptive to channel variations can not guarantee the delay requirements for the real life applications. To fulfill the QoS requirement of the 3GPP LTE-A standard, the control policy should be designed based on both channel condition and queue information of the users. By doing so, we can associate the users' traffic dynamics and channel variations with the SP's revenue. There are quite a number of works that considered the channel and queue information jointly and proposed a scheme for packet scheduling in OFDMA systems such as maximum-largest weighted delay (M-LWDF) [3], but most of them are not proper to use in the presence of the heterogeneous traffic since they do not provide bounded delay performance [4].

In this paper, an underlying information-theoretic principle is combined with a queuing-theoretic approach to achieve the guaranteed QoS for the users as well as the maximum revenue for the SP. We consider a pricing scheme that charges proportionally as the usage increases based on traffic type. The maximum achievable data rates by users are used as the revenue incentive for the SP. Two mathematical scheduling schemes are proposed under two different approaches, i.e., the MDP approach and the chance-constrained approach. Essentially, both proposed scheduling approaches assign each resource block to the best user and select the best corresponding MCS for each user while maximizing the overall system performance (e.g., SP's revenue) and guaranteeing the QoS requirement for heterogeneous services. Other than the nature of two approaches which achieve different objective goals, they are also different in terms of the complexity, the time between the decision epochs and the information required to make the decision.

Our first mathematical approach is based on constrained Markov decision process (CMDP) which maximizes the long-term average SP's revenue subject to long-term average queue length constraint. For this problem, we assume that channel state information (CSI) and queue state information (QSI) are available through feedback channels at the beginning of each decision epoch. The time between the decision epochs is as equal as the period of the feedbacked information. The Lagrangian dynamic programming approach is used to convert the constrained MDP to unconstrained MDP. The optimal control policy for unconstrained MDP is obtained by solving the well-known *Bellman's equation* using relative value iteration method. Then, using the concept of marginal delay cost, we show that the optimal policy obtained for the unconstrained MDP is also optimal for the CMDP. As our second approach, we propose a short-term optimization which maximizes the expected revenue and provides the short-term QoS provisioning for heterogeneous traffic using a set of stochastic constraints, e.g., chance constraints. Chance-constrained programming is one of the major approaches to deal with random parameters in the optimization area. The sources of randomness in our paper are the random arrival process and the random channel fading gains in the OFDMA system. We assume in the chance-constrained problem the probability distributions of the slow fading channel and the arrival process are known. Instead, the instantaneous CSI feedback from the users at each decision epoch is not required. The time between the decision epoches are considered to be many times of the decision epoch in the MDP approach, wherein between the decision epoches the slow fading channel process and the arrival process are still ergodic. To preserve the convexity and reduce the complexity of chance-constrained programming, we use Bernstein approximation [10] to obtain a conservative and deterministic approximation of the affine chance constraints.

The rest of the paper is organized as follows. Section 2 gives an overview of main LTE-A features followed by the system model. The scheduling problems proposed CMDP approach is formulated in Section 3. We propose the method to solve the CMDP problem in Section 4. In Section 5, the chance-constrained revenue optimization problem is formulated and the solution is proposed using Bernstein approximation. Section 6 presents the performance of our scheduling schemes. Finally, Section 6 draws the conclusions.

2 Model

In this section, the OFDMA downlink system model and queueing model are outlined. The simplified architecture of the LTE-A downlink packet scheduler in the eNodeB of the LTE-A system is shown in Fig. 1. At the beginning of each decision epoch, eNodeB receives CSI from the users and captures QSI by observing the users' buffer. The packet scheduler in eNodeB uses this information to make a decision based on the scheduling policy and passes it to the radio access unit. The technology to access the radio spectrum in downlink is OFDMA, which divides the bandwidth into a series of flat fading narrow bands [14]. The radio resource divisions of the LTE-A SP in time-frequency domains are shown in Fig. 2. One resource block corresponds to 180 kHz

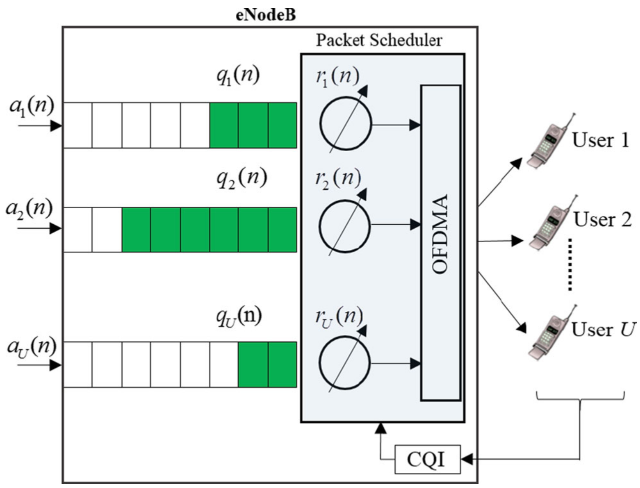

Fig. 1 Packet scheduling in the eNodeB of the LTE-A system

in the frequency domain and *0.5 ms* in the time domain. The minimum resource allocation unit is one scheduling block which is comprised of two consecutive resource blocks spanning a time duration of 1 ms known as transmit time interval (TTI). The update of the CSI, QSI and also the resource scheduling decision are carried out once every TTI. Here, the terms *TTI* and *time slot* are used interchangeably.

2.1 Physical layer

Consider a downlink OFDMA multiuser LTE-A system, let $\mathcal{U}$, $\mathcal{R}$, $\mathcal{M}$ and $\mathcal{P}$ be the sets of users, resource blocks, MCS schemes and prices, respectively. Define $U = |\mathcal{U}|$, $R = |\mathcal{R}|$, $M = |\mathcal{M}|$ and $P = |\mathcal{P}|$, where $|\cdot|$ represents the cardinality of a set. The channel between the eNodeB and any user $i \in \mathcal{U}$ is modeled as a frequency selective block fading channel, assuming that each resource block channel condition remains unchanged during a time interval of length TTI. At each TTI n, every user $i \in \mathcal{U}$ measures the signal to noise ratio (SNR) of the reference signals transmitted

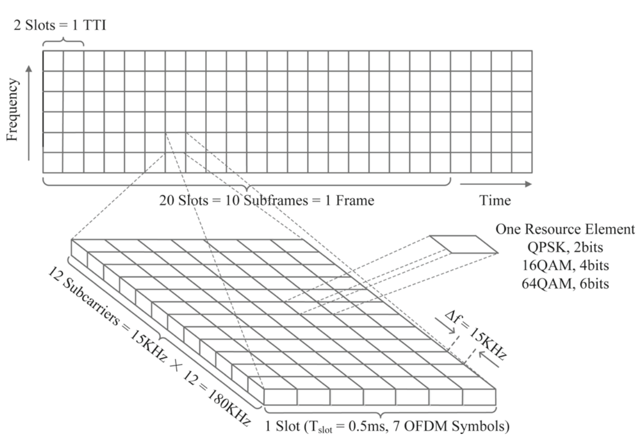

Fig. 2 Time-frequency resources in LTE-A

by the eNodeB over the channel, quantizes the SNR values and reports a channel quality indicator (CQI) vector $\mathbf{c}_i(n)$ to the eNodeB containing the $c_{ij}(n)$ values for all resource blocks $j \in \mathcal{R}$. Afterwards, eNodeB forms the CQI matrix $\mathbf{C}(n) = [\mathbf{c}_i(n)]$ and selects suitable set of MCS indexes corresponding to the $\mathbf{c}_i(n)$ to ensure a certain block error rate target (typically $< 10\ \%$) is met while achieving the highest transmit block size. Based on the 3GPP LTE-A standard, the scheduler should select a common MCS for each user over all resource blocks assigned to it at each TTI (refer to section 10.2 in [14]).

Denote $\mathbf{x}(n) = \{x_{ij}^m(n)\}$ as the resource block and MCS allocation strategy at TTI n, where $x_{ij}^m(n) = 1$ represents that resource block j is assigned to user i with MCS m at TTI n. Accordingly, denote $r_{ij}^m(n)$ as achievable data rate when $x_{ij}^m(n) = 1$. Further denote $\mathbf{r}(n) = (r_1(n), \cdots, r_U(n))$ as the achievable data rate of the users at TTI n, where $r_i(n)$ is

$$r_i(n) = \sum_{j\in\mathcal{R}} \sum_{m\in\mathcal{M}} r_{ij}^m(n) x_{ij}^m(n). \tag{1}$$

Consider $\mathcal{P}(n) = \{p_{ij}(n)\}$ as the unit prices set for all users $i \in \mathcal{U}$ over all resource blocks $j \in \mathcal{R}$. The pricing scheme assigns different costs per unit of usage for different types of traffic and charges proportionately as the usage increases [13]. Assuming that each user is associated with a single type of traffic, p_{ij} can be expressed by

$$p_{ij}(n) = \alpha_i r_{ij}(n), \tag{2}$$

where α_i is the constant coefficient to charge user i per unit of data rate and $r_{ij}(n) = \sum_{m\in\mathcal{M}} r_{ij}^m(n)$ is the amount of used data rate units for user i over resource block j at TTI n. Basically, $r_{ij}(n)$ is used as the revenue incentive for the SP. Denote extra auxiliary MCS assignment strategy $\mathbf{d} = \{d_i^m(n)\}$, where $d_i^m(n) = 1$ represents that user i chooses MCS m at TTI n based on the scheduling policy. We have the following widely-used assumption regarding the channel gains:

Assumption 1 The sequence of the fading channel variations follows an ergodic discrete time Markov chain [16]. It is also assumed that channel states are exactly known (or fully observed).

There is an overhead imposed by full channel acquisition over the uplink channel. MDP framework using partially observed channel states is out of scope of our MDP approach.

2.2 Source model and queue dynamics

In this paper, we adopt a queuing model such that each user has a queue in the eNodeB (see Fig. 1). Denote $\mathbf{q}(n) = (q_1(n), ..., q_U(n))$ to be the queue lengths of the users, where $q_i(n)$ represents the number of bits in the i-th user's queue. Further denote $\mathbf{a}(n) = (a_1(n), ..., a_U(n))$ to be the stochastic incoming traffic within the n-th time interval, where $a_i(n)$ and $\overline{a}_i$ represent the number of arrival bits and the average arrival rate to the i-th user's queue, respectively. We assume the packet length is fixed for different types of traffic. We put the following assumption for the incoming traffic:

Assumption 2 For all i and n, the random variables of the incoming traffic possess finite mean and finite variances; meanwhile, they are independent and identically distributed (i.i.d) over the decision epochs.

It is just for simplicity of our problem formulation that we assume i.i.d traffic arrival process. While, It is also possible to assume that traffic arrival is a Markov chain at the cost of further complexity.

In the n-th TTI, a batch of $\mathbf{a}(n)$ bits in the form of fixed-length packets arrive, followed by the departure of $\mathbf{r}(n)$ bits. We assume the incoming traffic $\mathbf{a}(n)$ are captured after the packet scheduler's decision at time n. The value of $\mathbf{a}(n)$ is endogenous parameter, whereas $\mathbf{r}(n)$ is exogenous and affected by the SP's action. Hence, the evolution of the queues can be written as

$$\mathbf{q}(n+1) = [\mathbf{q}(n) - \mathbf{r}(n)]^+ + \mathbf{a}(n), \tag{3}$$

where $[\cdot]^+$ is a componentwise operator defined as $\max\{0, \cdot\}$. We neglect the finite buffer size for the queue dynamics mentioned above.

3 Long-term revenue maximization problem under the MDP approach

In this section, we formulate the SP's revenue maximization problem. Consider $\mathcal{S}(n) = \{\mathbf{C}(n), \mathbf{q}(n)\}$ to be the system state space at TTI n, comprising the channel quality information and buffer state information. A policy is stationary if the decision rule is independent of the decision epochs. In our work, we assume a stationary and deterministic scheduling policy $\Omega = (\Omega_x, \Omega_d)$ which is a mapping function from system state space $s \in \mathcal{S}$ to the set of resource blocks and MCSs allocation action spaces, which are given by $\Omega_x(s) = \left\{x_{ij}^m \in \{0, 1\}, \forall i \in \mathcal{U}, j \in \mathcal{R}, m \in \mathcal{M}\right\}$ and $\Omega_d(s) = \{d_i^m \in \{0, 1\}, \forall i \in \mathcal{U}, m \in \mathcal{M}\big\}$, respectively.

The policy $\Omega = (\Omega_x, \Omega_d)$ should satisfy the practical constraints required by 3GPP LTE-A standard for all s. These constraints can be summarized as (i) each resource block can be assigned to only one user, (ii) each user can choose one MCS over all resource blocks assigned to it (iii) for each user the MCS is assigned only over those resource blocks assigned to the user, otherwise the related MCS indicator is assigned a value of zero (iv) the decision variables for resource blocks assignment and MCS assignment can take only the value of zero or one. With these physical constraints, the *per stage* constraint can be mathematically modeled as

$$\textstyle\sum_{i \in \mathcal{U}} \sum_{m \in \mathcal{M}} x_{ij}^m(n) \le 1, \qquad \forall j, n \tag{4}$$

$$\textstyle\sum_{m \in \mathcal{M}} d_i^m(n) \le 1, \qquad \forall i, n \tag{5}$$

$$x_{ij}^m(n) \le d_i^m(n), \qquad \forall i, j, m, n \tag{6}$$

$$x_{ij}^m(n), d_i^m(n) \in \{0, 1\}, \qquad \forall i, j, m, n. \tag{7}$$

We limit our policy space to unichain policies [5] and [6]. Given a unichain policy Ω, the induced Markov chain is ergodic and there exists a unique steady state distribution. Therefore, we have from the Little's theorem that the average delay of user i under policy Ω is given as

$$\overline{D}_i^{\Omega} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \frac{\mathbb{E}^{\Omega}[q_i(n)]}{\overline{a}_i}, \tag{8}$$

where $\mathbb{E}^{\Omega}$ is the expectation under the stationary policy Ω. Within the LTE-A network, the QoS requirements of heterogeneous services are classified to nine QoS class identifiers (QCI) based on their tolerable packet delay budgets and packet error loss rates (refer to Table 2.1 in [14]). For example web-browsing can tolerate delay up to 300 *ms* with maximum 10^{-6} packet loss rate. To consider different delay requirements associated with different QCIs, heterogeneous queue thresholds $\beta = \{\beta_i, \forall i\}$ are assigned to users that demand different services. For example, for the user which requires a service with tighter delay budget, we assign smaller β. To satisfy the long-term QoS requirements for different types of traffic we establish the long-term queue outage probability guarantee as follows:

$$\limsup_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} E^{\Omega}[I(q_i(n+1) > \beta_i)] \le \epsilon_i, \forall i. \tag{9}$$

where $I(\cdot)$ is the indicator function, which is equal to 1 if the condition inside the bracket holds and 0, otherwise. ϵ_i, which we refer to as the *desired queue outage probability*, is a small value representing the the maximum allowable loss for the violation of the queue threshold of user i.

$$\begin{aligned} &\mathrm{P}[s'|s, \Omega] = \\ &\mathrm{P}[s(n+1) \mid s(n){=}\{\mathbf{C}(n), \mathbf{q}(n)\}, \Omega(n)] = \\ &\mathrm{P}[\mathbf{C}(n+1) \mid \mathbf{C}(n)] \;.\; \mathrm{P}[\mathbf{q}(n+1)|\{\mathbf{C}(n), \mathbf{q}(n)\}, \Omega(n)]. \end{aligned} \tag{10}$$

Based on assumptions 1 and 2, the transition probabilities among the states can be written as Eq. 10, which due to the independence of incoming traffic among different users (10) can be further compactly written as

$$\begin{aligned}&\mathrm{P}[s'|s,\Omega]=\\&\mathrm{P}[\mathbf{C}(n+1)|\mathbf{C}(n)]\cdot\mathrm{P}\big[\mathbf{a}(n)=\mathbf{q}(n+1)-[\mathbf{q}(n)-\mathbf{r}(n)]^{+}\big]=\\&\mathrm{P}[\mathbf{C}(n+1)\mid\mathbf{C}(n)]\cdot\prod_{i}\mathrm{P}[a_i(n)].\end{aligned}\tag{11}$$

In our work, we are interested in finding the optimal policy, denote by Ω^*, such that the long-term average reward over an infinite time horizon is maximized subject to the per-stage (instantaneous) resource allocation constraints (4)-(7) and the queue outage probability constraint in Eq. 9. Mathematically, the problem is given by

$$\begin{aligned}\textbf{(P1):}\ \underset{\Omega}{\text{maximize}}\quad &\limsup_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}E^{\Omega}[\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}}(p_{ij}(n)\sum_{m\in\mathcal{M}}x_{ij}^{m}(n))]\\ \text{subject to}\quad &\text{constraints (4)}-\text{(7) and (9)}\end{aligned}$$

The SP's revenue is affected by the queue threshold value of different users. When the delay requirement of a user is more stringent (lower β), more resource blocks are seized by the user regardless of its possible bad channel quality over the seized resource blocks. This reduces the chance of assigning those resource blocks to users with better channel condition and incurs revenue loss for the SP.

Remark 1 Note that the optimal user scheduling problem with delay requirement **(P1)** is a constrained MDP in essence, which is widely used to deal with dynamic and multi-objective decision problems. Without constraint (9), **(P1)** can be easily resolved using value iteration or policy iteration method [12]. However, it becomes technically challenging since queue outage probability constraint in Eq. 9 may couple all the sequential decisions in addition to the per-stage resource constraints. In Section 4, we shall introduce the concept of *Marginal Delay Cost* as Lagrange multiplier, which can be proved to be efficient in solving **(P1)** with optimality guarantee under some conditions.

4 Marginal delay cost and the optimal scheduling policy

4.1 Marginal delay cost and optimality condition

In this section the optimal solution for **(P1)** is studied. We define the marginal delay cost for user i, which is denoted by λ_i, as the Lagrange multiplier for the delay constraint (9). Consider the following problem:

$$\begin{aligned}\textbf{(P2):}\ \underset{\Omega}{\text{maximize}}\quad &\limsup_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}E^{\Omega}\Big[\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}}p_{ij}(n)\sum_{m\in\mathcal{M}}x_{ij}^{m}(n)\\ &\qquad-\sum_{i\in\mathcal{U}}\lambda_i[I(q_i(n+1)>\beta_i)-\epsilon_i]\Big]\\ \text{subject to}\quad &\text{constraints (4)}-\text{(7) and } \lambda_i\geq 0\ \forall i.\end{aligned}$$

Let $R(s(n),\Omega)$ be the per stage reward that SP can achieve by choosing resource block and MCS allocation action under the policy Ω when the system state is s. Define the reward function at stage n as

$$R(s(n),\Omega)=\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}}p_{ij}(n)\sum_{m\in\mathcal{M}}x_{ij}^{m}(n)-\sum_{i\in\mathcal{U}}\lambda_i[I(q_i(n+1)>\beta_i)-\epsilon_i].\tag{12}$$

Note that the optimal user scheduling problem **(P2)** with marginal delay cost λ_i is equivalent to the Lagrangian function of problem **(P1)** after incorporating the queue outage probability constraint (9) into the objective function. Furthermore, we have the following lemma to show the relationship between **(P1)** and **(P2)**.

Lemma 1 *Let Ω^* and λ_i be the optimal policy and the marginal delay cost for problem **(P2)**. If the queue outage probability constraint (9) is strictly binding with policy Ω^*, then the policy Ω^* is also optimal for problem **(P1)** with λ_i serving as the corresponding optimal Lagrange multiplier.*

The proof can be followed from the subgradient method, which is omitted in this paper for brevity. Note that if β_i is sufficiently large, such that the queue outage probability constraint (9) for user i is slack at the optimum of problem **(P1)**, the value of $\lambda_i=0$. This can be trivially understood by that the user can tolerate large delay such that the SP does not need to consider any delay cost. When β_i is not sufficiently large, the value of λ_i can be determined by using the bisection method, which can iteratively reach the specified average delay β_i. Denote $\Omega^*_{\lambda_i}$ as the optimal scheduling policy for a given Lagrangian multiplier λ_i. The queue outage probability for user i under the scheduling policy $\Omega^*_{\lambda_i}$ is given by:

$$Q_i^{\Omega^*_{\lambda_i}}=\limsup_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}E^{\Omega^*_{\lambda_i}}[I(q_i(n+1)>\beta_i)],\forall i.\tag{13}$$

It was proved in [2] that $Q_i^{\Omega^*_{\lambda_i}}$ is a piecewise linear non-increasing function of λ_i. We can find the optimal Lagrange multiplier λ_i^* through the following update:

$$\lambda_i^{l+1} = \max(\lambda_i^l + \gamma^l (Q_i^{\Omega^*_{\lambda_i^l}} - \epsilon_i), 0), \quad (14)$$

where $\gamma_i^l = \frac{1}{l}$ and l is the iteration index. The convergence to λ_i^* is ensured due to the fact that $Q_i^{\Omega^*_{\lambda_i}}$ is a piecewise convex function of Lagrange multiplier λ_i. Later on in this paper, we consider that the marginal delay cost is predetermined to reach a certain queue outage probability. By doing so, the following two advantages facilitate exact performance analysis and the practical implementation of this scheduling policy:

– The existence of an optimal policy that is both deterministic and stationary for Problem **(P2)** has already been shown in [5], which significantly reduces the implementation complexity.
– The marginal delay cost λ_i represents the delay sensitivity of the traffic of user i. By using the fixed λ_i, we can evaluate how the delay sensitivity of user traffic influences the optimal scheduling policy of the SP. Moreover, we can analyze the impact of multiple users with heterogeneous queue outage probabilities on the optimal scheduling policy.

4.2 Optimal scheduling policy

The structure of our MDP problem can be expressed as follows:

– State: $\mathcal{S} = \{\mathbf{C}, \mathbf{q}\}$
– Action: $\{x_{ij}^m \in \{0, 1\}\}, \{d_i^m \in \{0, 1\}\}$
– Reward: R is given in Eq. 12.

– Transition probability matrix: $\mathrm{P}[s'|s, \Omega]$ is given in Eq. 11.

Per stage constraints in Eqs. 4-6 are applied to the action space. The channel variations across the users provides the opportunity for the SP to increase its revenue by using multiuser diversity gain. However, the SP should make sure that it satisfies the desired queue outage probabilities of different users. When the SP applies a stationary policy Ω, the induced Markov chain is recurrent and the optimal long-term average sum revenue is independent of the initial state. Under the unichain policy assumption, there exists an optimal control policy Ω^* for the problem **(P2)**, such that for any state $s \in \mathcal{S}$ the following *Bellman's equation* is satisfied [12]:

$$J + V^*(s) = \max_{\Omega} \left\{ R(s, \Omega) + \sum_{s' \in \mathcal{S}} \mathrm{P}[s' \mid s, \Omega] V(s') \right\}, \forall s \in \mathcal{S} \quad (15)$$

where $V^*(s)$ is the optimal value function for state s and $R(s, \Omega)$ is the reward function defined in Eq. 12. J is the optimal reward per stage which shows the maximum revenue subject to the given queue outage probability constraint. With the stationarity assumption, time index n is eliminated. The *Bellman's equation* can be derived numerically through *Relative Value Iteration* algorithm which can be written as [12].

$$V_n(s^0) + V_{n+1}(s) = \max_{\Omega} \left\{ R(s, \Omega) + \sum_{s' \in \mathcal{S}} \mathrm{P}[s' \mid s, \Omega] V_n(s') \right\}, \forall s \in \mathcal{S} \quad (16)$$

where s^0 is any fixed state. Note that in the proposed MDP problem the desired queue outage probability for each user is guaranteed in a long-term manner by Eq. 9. Besides, since the MDP-based scheduling problem requires the instantaneous CSI to be reported by the users, there is a large overhead over the uplink channel. In the next section, we propose a novel chance-constrained based problem which maximizes the short-term expected revenue while providing the stochastic short-term QoS guarantee. Moreover, this approach performs the decision making regardless of the instantaneous CSI.

5 Short-term revenue maximization problem under the chance-constrained approach

In this section, we formulate the optimization problem of the SP's revenue based on stochastic chance constraint on queue length of the users. Since the queue state of the users vary in a slower time scale than the instantaneous channel state of the users, we consider the slow fading channel, in which the source of randomness in the SNR value is from long-term channel variations, i.e., path loss and shadowing effects. Denote t as the time window, in which within that the slow fading channel process and arrival process are considered to be ergodic. Based on slow adaptive channel characteristic and packet arrival process, t ought to span the length of many time slots n. Consider the channel gain g_{ij} for user i at resource block j to follow a general probability density function (PDF) $f_{g_{ij}}(\xi)$. The achievable data rate $r_{ij}^m(n)$ for user i over resource block j at time n can be expressed as

$$r_{ij}^m(n) = W \log_2(1 + \frac{p g_{ij}}{\Delta_m \sigma^2}) \quad (17)$$

where σ^2 is power of additive white Gaussian noise as background noise, W is the bandwidth of a single resource block, and Δ_m is the capacity gap for the bit error rate and MCS m. The maximum supportable MCS by each user is determined according to channel condition using the rate adaptation techniques.

The stochastic QoS guarantee for users with heterogeneous service classes can be expressed as a chance constraint on queue length of the users as follows

$$\Pr\left\{[q_i(n) - \sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}} r_{ij}^m(n)x_{ij}^m(t)]^+ + a_i(n) > \beta_i|q_i(n)\right\} \leqslant \epsilon_i \forall i, \forall n \in t. \tag{18}$$

where $x_{ij}^m(t)$ is the binary decision variable for the given time window t. For the given queue backlog $q_i(n)$, Eq. 18 guarantees a bounded queue outage probability (less than ϵ_i) which is the probability that for each user i during the time window t the queue length to be beyond of its predefined threshold β_i. Now, we can model the revenue maximization problem for each time window t as follows

$$\begin{aligned}
\textbf{(P3):}\ \underset{\mathbf{x},\mathbf{d}}{\text{maximize}}\quad & \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}} E[p_{ij}(n)]\sum_{m\in\mathcal{M}} x_{ij}^m \\
\text{subject to}\quad & \text{constraint (18)} \\
& \sum_{i\in\mathcal{U}}\sum_{m\in\mathcal{M}} x_{ij}^m \leq 1, \qquad \forall j \qquad (19)\\
& \sum_{m\in\mathcal{M}} d_i^m \leq 1, \qquad \forall i \qquad (20)\\
& x_{ij}^m \leq d_i^m, \qquad \forall i, j, m \quad (21)\\
& x_{ij}^m, d_i^m \in \{0, 1\}, \qquad \forall i, j, m. \quad (22)
\end{aligned}$$

where in $E[p_{ij}(n)]$ the time average revenue over the time window t is replaced by the ensemble average due to ergodicity of the slow fading channel within the time window t. d_i^m is the MCS decision variable for the given time window t. The chance constraint (18) in **(P3)** makes the optimization highly intractable due to difficulty of verifying its convexity. In this work, Bernstein approximation [10] is used to obtain a conservative convex approximation of the affine chance constraint in Eq. 18. Bernstein approximation is a recent advance in the field of chance-constrained programming that provides a tractable conservative deterministic and convex approximation for the chance constraint [17].

Proposition 2 *The stochastic queue-constrained problem in **(P3)** can be approximated by the deterministic and convex optimization problem defined as*

$$\begin{aligned}
\textbf{(P4):}\ \underset{\mathbf{x},\mathbf{d}}{\text{maximize}}\quad & \textstyle\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}} E[p_{ij}(n)]\sum_{m\in\mathcal{M}} x_{ij}^m \\
\text{subject to}\quad & \inf_{\varrho_i>0}\{\Psi_i(\mathbf{x},\zeta_\mathbf{i}) - \varrho_i \log\epsilon_i\} \leq 0,\ \forall i \qquad (23)\\
& \text{constraints (19)} - (22)
\end{aligned}$$

where $\Psi_i(\mathbf{x},\zeta_\mathbf{i}) = q_i + \varrho_i \sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}} \Lambda_{r_{ij}^m}(-\varrho_i^{-1}x_{ij}^m) + \varrho_i\Lambda_{a_i}(\varrho_i^{-1}) - \beta_i$, $\mathbf{x} = \{x_{ij}^m\}$ and $\zeta_\mathbf{i} = (\mathbf{r}_i, a_i)$. Note that we denote $\Lambda_{r_{ij}^m}$ and Λ_{a_i} as the cumulant (log-moment) generating function of the data rate and the traffic arrival process, respectively.

Proof See Appendix. □

Proposition 3 *The approximated chance constraints in Eq. 23 are convex with respect to decision variable* $\mathbf{x}$.

Proof The function inside $\inf_{\varrho>0}(\cdot)$ is positive weighted summation of two functions with respect to $\mathbf{x}$. Both of these functions are convex, since the log-moment generating function is convex. The inf operator over $\varrho > 0$ outside the discussed expression also preserves the convexity.

According to [10], the chance constraint in Eq. 18 holds if there exists a $\varrho_i > 0$ satisfying constraint (23) in **(P4)**. Note that Problem **(P4)** is a convex optimization problem which has a convex subproblem that requires to minimize the function $\Psi_i(\mathbf{x},\zeta_\mathbf{i}) - \varrho_i\log\epsilon_i$ over ϱ_i. According to Eq. 23, $\Psi_i(\mathbf{x},\zeta_\mathbf{i})$ is convex and differentiable over ϱ_i. Therefore, it is always easy to obtain the minimum of function $\Psi_i(\mathbf{x},\zeta_\mathbf{i}) - \varrho_i\log\epsilon_i$ by setting the first derivative to be 0. In the following lemma, the derivative of constraint (23) is obtained. □

Lemma 4 *The first derivative of* $\Theta_i = \Psi_i(\mathbf{x},\zeta_\mathbf{i}) - \varrho_i\log\epsilon_i$ *with respect to* ϱ_i *can be written as Eq. 24.*

$$\begin{aligned}
\frac{\mathrm{d}\Theta_i}{\mathrm{d}\varrho_i} &= \sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}} \Lambda_{r_{ij}^m}(-\varrho_i^{-1}x_{ij}^m) + \Lambda_{a_i}(\varrho_i^{-1}) - \frac{\bar{a}_i\exp(\frac{1}{\varrho_i})}{\varrho_i\ln(10)} - \log(\epsilon_i)\\
&+\varrho_i\sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}}\frac{\log\int_0^\infty \ln(1+\frac{p\xi}{\Delta_m\sigma^2})^{1+\frac{-Wx_{ij}^m}{\varrho_i\ln(2)}}(\frac{Wx_{ij}^m}{\mu_i\varrho_i^2\ln(2)})\exp(-\frac{\xi}{\mu_i})\mathrm{d}\xi}{\ln(10)\int_0^\infty(1+\frac{p\xi}{\Delta_m\sigma^2})^{\frac{Wx_{ij}^m}{\varrho_i\ln(2)}}\frac{1}{\mu_i}\exp(-\frac{\xi}{\mu_i})\mathrm{d}\xi}.
\end{aligned} \tag{24}$$

We can determine the feasibility of the given $\mathbf{x}$ through finding the optimal solution of Eq. 23, i.e., $\varrho_i^\star$ using Lemma 4. If $\varrho_i^\star > 0$ and $\Theta^\star < 0$, then constraint (23) is feasible for the given $\mathbf{x}$.

6 Performance evaluation

In this section, the performance bound of the proposed scheduling problems, i.e. the MDP and the chance-constrained approaches are evaluated in terms of SP's

Table 1 Implementation parameters

Parameter	Value
Radius of cell	250m
Number of users	3
Traffic model	Backlogged traffic model
Path loss	$58.1 + 37.6 \log_{10}(d_{\text{km}})$
Lognormal shadowing	$\mathcal{N}(0, 8\text{db})$
Base stations TX power	30 Watts
Thermal noise power	1 Watt
Total no. of RBs	6
CQI report period	1 ms
W	180 Hz
Frequency granularity for CQI	One RB
ARQ process	Zero transmission attempt
TTI	1 ms
time window t	200 ms

revenue and queue outage probability for LTE-A downlink system with heterogeneous traffic types. The proposed schemes are implemented in MATLAB. We assume a single hexagonal cell, whereas users do not leave it within the simulation. The transmission power is 30 Watts and the power allocation across the RBs is homogeneous. For the MDP approach, we utilize the 3GPP typical urban channel model [1] to generate the required SNR values and accordingly, create the CQI state space for our model. Without loss of generality, we assume the *frequency granularity* of the CQI measurement to be one RB and the period of the CQI reporting to one TTI. Then, the maximum supportable MCS for each user over each resource block is obtained based on a mapping from CQI reports to ensure a block error rate target less than 10 %. For the chance-constrained approach, no feedback scheme is considered. The slow fading channel characteristics for the chance-constrained approach are given in Table 1. The small-scale channel fading is assumed to be Rayleigh fading distributed. Three MCS levels are available for transmission. The minimum required SNR to determine the maximum supportable MCS level is presented in Table 2. The decision epochs for the MDP-approach are set to be equal to the same time duration of the CQI reports, e.g. $n = 1$ms. For the chance-constrained approach the decision epochs are each $t = 200$ms which is 200 times of the MDP-approach decision epochs. The simulation settings used in our implementation are summarized in Table 1. The total number of users is considered to be 3, unless otherwise noted. The small sized network is opted to avoid creating of a large dimension state space in the MDP approach. For the chance-constrained approach there is no restriction to increase the number of the users, since it is solved in polynomial time with respect to the size of users, resource blocks and MCSs.

Table 2 Minimum SNR required for different MCSs

MCS level	Modulation	Coding	Minimum SNR (db)
1	BPSK	1/2	3.0
2	QPSK	3/4	8.0
3	64QAM	3/4	13.5

The network includes maximum of three users which can be grouped into two classes, i.e., Class I and Class II. These two classes are heterogeneous in terms of arrival rates, queue thresholds (delay requirements) and queue outage probabilities. Class I and class II are characterized by 3-tuple of $(\bar{a}, \beta, \epsilon) = \{(0.1$ pkts/TTI, 120kbits, $0.05), (0.2$ pkts/TTI, 50 kbits, $0.005)\}$, unless otherwise noted. The packets arrive at random according to the Poisson process. A user from class I represents a less delay-sensitive user, while a user from class II represents a user with more stringent delay requirements. Without loss of generality, let's assume SP acquires half unit of currency (0.5 USD) for every 1 kbits of class I traffic and one unit of currency (1 USD) for every 1 kbits of class II transmitted data. In the following subsections, the results for the MDP-approach and the chance-constrained approach are presented.

6.1 Performance evaluation for the MDP approach problem

In the first test, the SP's revenue is evaluated for different combinations of users of different classes. The results presented in Table 3 show that generally by increasing the number of users, e.g., compare row 1 against other rows, the system achieves higher revenue. However, the improvement in revenue is less when a user with more stringent delay requirement is added to the system, e.g. row 3 achieves less revenue than row 2. We also compare the SP's revenue for the case of three users of class I with the case of three users of class II. As it is shown in rows 3 and 6 in Table 3, SP's revenue is higher when users have more relaxed desired queue outage probability in the system.

The queue outage probabilities for different traffic mix are compared in Table 4. The result in row 1 shows a zero outage probability when there is a single class I user in the system. When the number of users with stringent delay requirement increases, it results in higher queue outage probability as shown in rows 2 and 3.

Table 3 SP's revenue for different numbers of users

	Number of the users (class I, class II)	Average total SP's revenue (USD)
1	(1, 0)	5.1875
2	(2, 0)	8.5417
3	(3, 0)	10.3594
4	(2, 1)	8.918
5	(0, 2)	7.593
6	(0, 3)	8.0218

Next, we examine the revenue under different channel qualities for two classes of users. The channel quality of the users over all the states $\mathcal{S}$ is quantized to 5 levels, in which channel quality 5 is the best. The average total revenues over different channel quality levels are compared when there are *case1*: (1,1), *case2*: (2,0) and *case3*: (0,2) number of class I and class II users, respectively. The results in Fig. 3 show that when the channel condition improves, SP's revenue increases as well. However, the improvement in the revenue is higher when delay requirement of the users is less stringent.

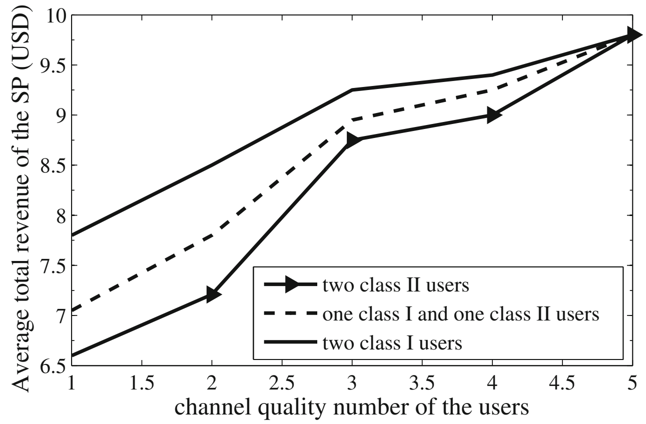

Fig. 3 Average total revenue of the SP for different channel qualities

6.2 Performance evaluation for the chance-constrained approach problem

In this part, we investigate the performance of the chance-constrained problem in **(P4)** using the parameter settings stated in the beginning of this section. Each packet size is 100 bits for both service classes. We suppose the path-loss and shadowing do not vary within the decision epoch of $t = 200$ms. Our scheduling scheme is evaluated using the Monte-Carlo based simulation. We conducted the experiments over 100 independent windows of length t each.

Denote A_i as the required available space of a queue, which is obtained as the difference of the queue threshold and the queue state (i.e., $A_i = \beta_i - q_i, \ \forall i$). In our first set of evaluations for the chance-constrained based problem, we test the effect of arrival rates on the feasibility set defined by constraint (23) for two cases: when constraint (23) (i) imposes the feasibility set of **(P4)** to be empty. (ii) does not shrink the feasibility set of **(P4)**. In the former one, A_i represents the minimum required available space of the queues or, in other words, the maximum queue length of the users that would trigger packet drop due to queue outage. While in the latter one, A_i represents the minimum required available space of the queues or, in other words, the maximum allowable queue length of the users, that satisfies constraint (23) without reducing the feasibility set. We have evaluated A_i under a simple scenario where user 1, user 2 and user 3 are associated with class I, class II and class II, respectively. In Fig. 4, the minimum A_2 value for case (i) is found exhaustively when only user 2's queue is granted to be tuned. While in Fig. 5, the minimum required A_1, A_2 and A_3 values for case (ii) are exhaustively obtained when queues of user 1 and user 2 are tuned. Different colors represent the minimum required available space for three users.

Table 4 Queue outage probability for different numbers of users

	Number of the users (class I, class II)	Queue outage probabilities (class I, class II)
1	(1, 0)	(0, –)
2	(2, 0)	(0.04, –)
3	(0, 2)	(–, 0.0081)
4	(2, 1)	(0.0521, 0.0081)

We now compare the average revenue achieved by the chance-constrained short-term scheduling problem in **(P4)** against that of an instantaneous revenue maximization scheduling, in which resource blocks and MCS are assigned each 1ms under zero queue outage probability. In instantaneous scheduling problem under zero queue outage probability, the size of feasibility set is affected by two factors: (i) at each decision epoch, the scheduling decision needs to ensure the zero queue outage probability for the subsequent time slot (1ms), instead of ensuring a desired queue outage probability for $t = 200$ms. This factor expands the set of feasible solutions. (ii) since the queue outage probability is zero in instantaneous scheduling problem, the scheduling decision should 100 % guarantee the queue length of all users is not beyond the their predefined thresholds. This factor shrinks the set of feasible solutions. Figure 6 depicts the accumulated revenue for instantaneous scheduling problem under zero queue outage probability compared to the average revenue for **(P4)** within 2000ms. As expected, the instantaneous scheduling achieves relatively better revenue

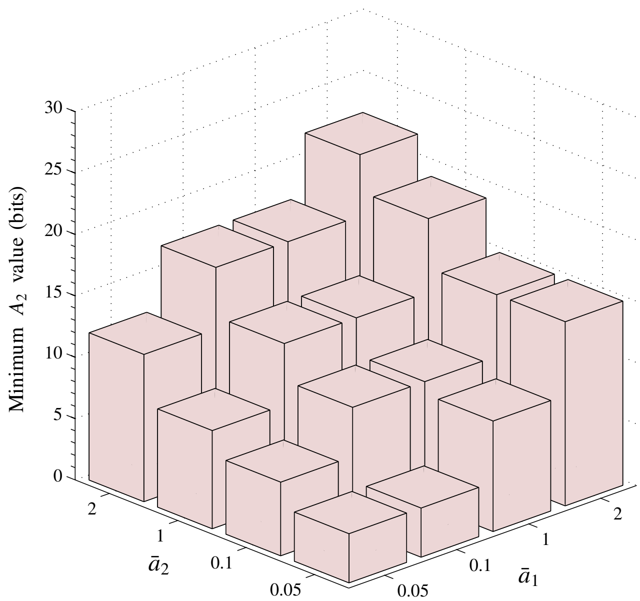

Fig. 4 The minimum required available space of user 2's queue to obtain a non-empty feasibility set

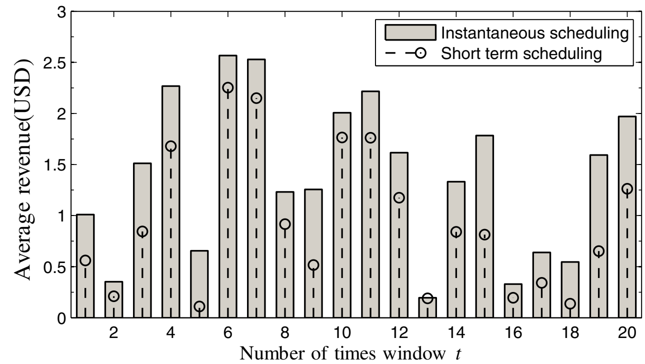

Fig. 6 Comparison of the average revenue(USD) between the instantaneous and short term schedulings

(on average 33 %) compared to the short term scheduling in **(P4)**. Considering the parameter setting that each short term scheduling epoch spans the length of 200 instantaneous decision epochs, the control signalling overhead and computational complexity for short term scheduling are 1/200 of that for instantaneous scheduling problem, trading off 33 % of average revenue for it seems reasonable.

In Fig. 7, we measure the queue outage probability which is defined before as the probability that queue length for users i is beyond its predefined threshold β_i. At each time window t, the slow fading channel parameter μ_i is generated based on given model in Table 1. The users' locations are generated using Poisson point process and two positions are picked up as locations of two users to obtain d_{km} for the path-loss modelling. User 1 and user 2 are associated with class I and class II, respectively. As it is shown in Fig. 7, the queue outage probabilities for both classes are always smaller than the their desired queue outage probabilities. The reason is the conservative approximation of the chance constraint using Bernstein approximation. The space between the obtained queue outage probabilities and the desired queue outage probabilities can be reduced by setting the predefined thresholds to larger values than the desired ones. To illustrate the effect of desired queue outage probabilities, we evaluate the optimal objective value in

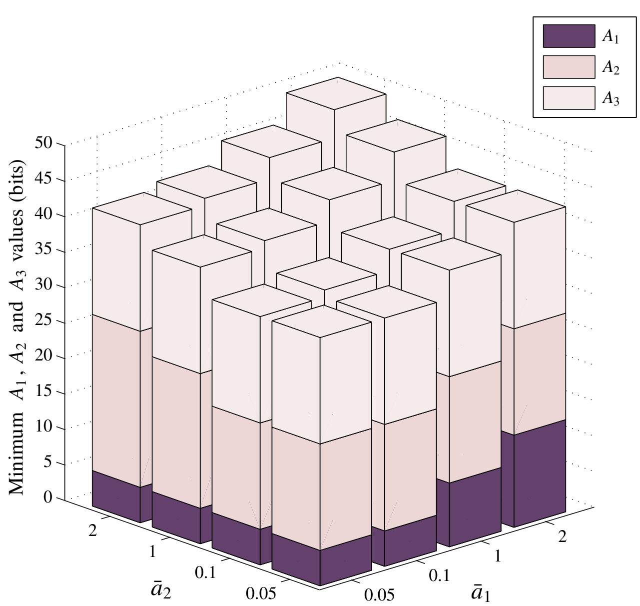

Fig. 5 The minimum required available spaces for all the users' queues to satisfy constraint (23) without shrinking the feasibility set

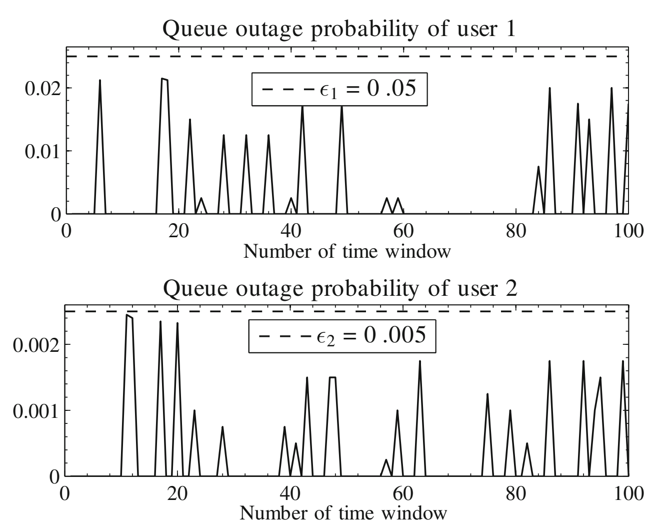

Fig. 7 Queue outage probabilities over 100 time windows t

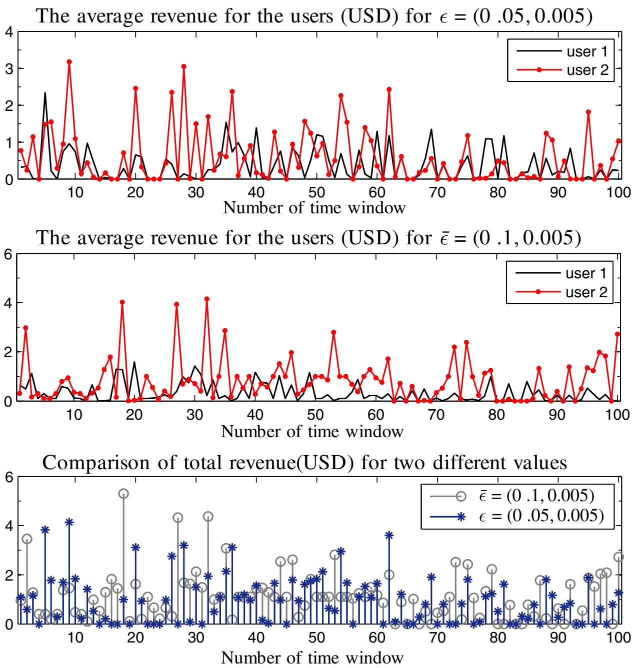

Fig. 8 Average revenue for different queue outage probabilities over 100 time windows t

(P4) for $\epsilon = [0.05, 0.005]$ and $\bar{\epsilon} = [0.1, 0.005]$ along the time domain, in Fig. 8. $\bar{\epsilon}$ is the maximum allowable queue outage probability that still ensures the queue outage probability lower than users' desired queue outage probability, i.e., ϵ. **(P4)** achieves a higher revenue by allowing the upper bound of the queue outage probability to be large, which means less stringent delay constraints for the users. Indeed, by comparing the averaged obtained revenues under ϵ and $\bar{\epsilon}$ in Fig. 8, we observe that the latter one achieves 15.86 % less revenue in comparison to the former one.

7 Concluding remarks

In this paper, we proposed two scheduling schemes which utilize the MDP and the chance-constrained optimizations, to maximize the long-term and short-term LTE-A SP's revenues subject to the long-term and short-term heterogeneous QoS constraints of the users, respectively, as well as satisfying the resource scheduling constraints of the LTE-A system according to the 3GPP LTE-A standard. Other than optimizing different time-scale objectives, the two approach are also different in terms of the complexity, the time between the decision epochs and the information required to make the decision. The key contribution in the short-term scheduling is to use the Bernstein approximation to transform the chance constraint to a convex, deterministic and computationally tractable constraint so that large-sized problems can be solved.

Appendix

Proof of proposition 2

To apply the Bernstein approximation for the constraint in Eq. 18, based on our commonly used assumption of backlogged traffic model we can omit the $[\cdot]^+$ operator form the constraint. In the backlogged traffic model, as the name suggests, each user has always packet to transmit. Accordingly, the inequality $q_i(n) - \sum_{j\in\mathcal{R}} \sum_{m\in\mathcal{M}} r_{ij}^m(n) x_{ij}^m + a_i(n) > \beta_i$ can be equivalently expressed as

$$F_i(\mathbf{x}, \zeta_{\mathbf{i}}) > 0, \tag{25}$$

where $\zeta_{\mathbf{i}} = (\mathbf{r}_i, a_i)$ and

$$F_i(\mathbf{x}, \zeta_{\mathbf{i}}) \triangleq q_i(n) - \sum_{j\in\mathcal{R}} \sum_{m\in\mathcal{M}} r_{ij}^m(n) x_{ij}^m + a_i(n) - \beta_i. \tag{26}$$

$F_i(\mathbf{x}, \zeta_{\mathbf{i}})$ is in the form of affine chance constraint which involves linear form of the random variables $\zeta_{\mathbf{i}} = (\mathbf{r}_i, a_i)$. Based on the Bernstein approximation, constraint (18) can be approximated by

$$\inf_{\varrho_i > 0} \{\Psi_i(\mathbf{x}, \zeta_{\mathbf{i}}) - \varrho_i \log \epsilon_i\} \le 0, \forall i \tag{27}$$

where

$$\begin{aligned} \Psi_i(\mathbf{x}, \zeta_{\mathbf{i}}) &= \varrho_i \log \mathbb{E}[\exp(\varrho_i^{-1}(F_i(\mathbf{x}, \zeta_{\mathbf{i}}))] \\ &= q_i + \varrho_i \sum_{j\in\mathcal{R}} \sum_{m\in\mathcal{M}} \Lambda_{r_{ij}^m}(-\varrho_i^{-1} x_{ij}^m) + \varrho_i \Lambda_{a_i}(\varrho_i^{-1}) - \beta_i, \end{aligned} \tag{28}$$

where $\Lambda_{r_{ij}^m}$ and Λ_{a_i} are the cumulant (log-moment) generating function of the data rate and arrival process, respectively.

In the sequel, we derive the cumulant generating function of the random variable r_{ij}^m, which is a function of the channel gain random variable. The moment generating function (MGF) of $r(\gamma_{ij}^m) = W \log(1 + \gamma_{ij}^m)$, where $\gamma_{ij}^m = \frac{p g_{ij}}{\Delta_m \sigma^2}$, is given in [7] as follows

$$M_{r_{ij}^m}(y) = \mathbb{E}[\exp(-y r(\gamma_{ij}^m))] = 1 + \int_0^{+\infty} Q(\ln(2)y, \xi) M_{\gamma_{ij}^m}^{(1)}(\xi) \mathrm{d}\xi, \tag{29}$$

where $Q(a, u) = \Gamma(a, u)/\Gamma(a)$ is the regularized Gamma function[1] and $M_{\gamma_{ij}^m}^{(1)}(\cdot)$ is the first derivative of the MGF of γ_{ij}^m. $M_{\gamma_{ij}^m}(\cdot)$ is known in closed-form for many fading distributions [15]. Without loss of generality, consider the channel gain as an exponentially distributed random variable with PDF given by $f_{g_{ij}}(\xi) = \frac{1}{\mu_i} \exp(-\frac{\xi}{\mu_i})$, where μ_i is the mean slow fading channel gain for user i and can be

[1] $\Gamma(a, u) = \int_u^\infty t^{a-1} e^{-t} \mathrm{d}t$ and $\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} \mathrm{d}t$

characterized based on path-loss and shadowing effects. In practice, the difference between different resource blocks is indistinguishable for slow fading channel parameter μ_i. The value of μ_i oughts to be updated at the beginning of each time window t. Hence, the cumulant generating function of the data rate can be achieved as

$$\Lambda_{r_{ij}^m}(y) = \log\left[\int_0^\infty (1+\frac{p\xi}{\Delta_m\sigma^2})^{\frac{W_y}{\ln 2}}.\frac{1}{\mu_i}\exp(-\frac{\xi}{\mu_i})d\xi\right]. \tag{30}$$

To calculate the cumulant generating function of the arrival process, e.g., Λ_{a_i}, without loss of generality consider the arrival process for the traffic of the user i follows Poisson distribution with parameter as $f_{a_i}(k) = \frac{\bar{a}_i^k e^{-\bar{a}_i}}{k!}$, where $\bar{a}_i$ represents the average rate. Λ_{a_i} can be computed as

$$\Lambda_{a_i}(y) = \log\mathbb{E}[e^{ya_i}] = \log(e^{\bar{a}_i(e^y-1)}) = \frac{\bar{a}_i(e^y-1)}{\ln 10}. \tag{31}$$

References

1. 3GPP, TR 25.943, v10.0.0 (2011-04): Technical specification group radio access networks; deployment aspects (release 10) http://www.etsi.org/deliver/etsi_tr/125900_125999/125943/11.00.00_60/
2. Altman E (1995) Constrained Markov decision processes
3. Andrews M, Kumaran K, Ramanan K, Stolyar A, Whiting P, Vijayakumar R (2001) Providing quality of service over a shared wireless link. IEEE, Commun Mag 39(2):150–154. doi:10.1109/35.900644
4. Balakrishnan R, Canberk B (2014) Traffic-aware QoS provisioning and admission control in ofdma hybrid small cells. IEEE Trans, Veh Technol 63(2):802–810. doi:10.1109/TVT.2013.2280124
5. Bertsekas D (1987) Dynamic Programming: Deterministic and Stochastic Models, Pearson Higher Education & Professional Group. http://books.google.com.hk/books?id=-6RiQgAACAAJ
6. Cui Y, Lau VKN (2010) Distributive stochastic learning for delay optimal OFDMA power and subband allocation. IEEE Transaction of Signal Procssing 58(9):4848–4858. doi:10.1109/TSP.2010.2050062
7. Di Renzo M, Graziosi F, Santucci F (2010) Channel capacity over generalized fading channels: A novel MGF-Based approach for performance analysis and design of wireless communication systems. IEEE Trans Veh Technol 59(1):127–149. doi:10.1109/TVT.2009.2030894
8. Kokku R, Mahindra R, Zhang H, Rangarajan S (2013) Cellslice: Cellular wireless resource slicing for active RAN sharing. In: Communication Systems and Networks (COMSNETS), 2013 Fifth International Conference on, pp 1–10. doi:10.1109/COMSNETS.2013.6465548
9. Lee SB, Choudhury S, Khoshnevis A, Xu S, Lu S (2009) Downlink MIMO with frequency-domain packet scheduling for 3GPP LTE. In: IEEE INFOCOM, pp 1269 –1277. doi:10.1109/INFCOM.2009.5062041
10. Nemirovski A, Shapiro A (2006) Convex approximations of chance constrained programs. SIAM J Optim 17:969–996
11. Niafar S, Huang Z, Tsang DHK (2014) An optimal standard-compliant MIMO scheduler for LTE downlink. IEEE Trans Wirel Commun 99:1–10. doi:10.1109/TWC.2014.012814.131600
12. Powell W (2007) Approximate Dynamic Programming: Solving the Curses of Dimensionality. Wiley Series in Probability and Statistics. Wiley. http://books.google.com.hk/books?id=WWWDkd65TdYC
13. Ruby R, Leung V (2011) Towards QoS assurance with revenue maximization of LTE uplink scheduling. In: Communication Networks and Services Research Conference (CNSR), 2011 Ninth Annual, pp 202–209. doi:10.1109/CNSR.2011.37
14. Sesia S, Toufik I, Baker M (2011) LTE-The UMTS Long Term Evolution: From Theory to Practice. Wiley. http://books.google.com.hk/books?id=beIaPXLzYKcC
15. Simon M, Alouini M (2000) Digital communication over fading channels: a unified approach to performance analysis. Wiley series in telecommunications and signal processing. Wiley. http://books.google.com.hk/books?id=CvhSAAAAMAAJ
16. Tan C, Beaulieu N (2000) On first-order Markov modeling for the Rayleigh fading channel. IEEE Trans Commun 48(12):2032–2040. doi:10.1109/26.891214
17. Xu W, Tajer A, Wang X, Alshomrani S (2014) Power allocation in MISO interference channels with stochastic CSIT. IEEE Trans Wirel Commun 13(3):1716–1727. doi:10.1109/TWC.2014.012814.131600
18. Zhang H, Prasad N, Rangarajan S (2012) MIMO downlink scheduling in LTE systems. In: IEEE INFOCOM, pp 2936 – 2940