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Cooperative Distributed Energy Generation and Energy Trading for Future Smart Grid

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Abstract: In this paper, we investigate the cooperative distributed energy generation and energy trading for future smart grid. In our model, a group of energy users, who are equipped with the capabilities of distributed energy generation, are allowed to trade energy in a cooperative manner with the goal to minimize their total energy-provisioning cost while meeting the local demand of each individual energy user. Moreover, each user also expects to benefit from the cooperative energy generation and trading with the others. Motivated by these objectives, we first jointly determine the optimal energy scheduling decisions for all energy users such that their total energy-provisioning cost can be minimized. Then, based on the optimal energy scheduling decisions, we further determine the optimal transaction costs associated with the users' energy trading to ensure that each of them can positively benefit from the cooperation. Extensive numerical results are provided to show the advantages of the proposed cooperative energy generation and trading model as well as our proposed algorithms to achieve the optimal solutions.

Key Words: Distributed Energy Generation, Energy Trading, Cooperative Optimization, Transaction Cost

1 Introduction

Smart grid (SG) has been widely conceived to be able to increase the operational reliability, efficiency and security of future power grid [1]. An important feature of smart grid is its advanced information and control technologies, which enable a real-time demand-response management mechanism, such that both the energy consumers and generators can adapt their energy scheduling decisions in an intelligent and efficient manner. Meanwhile, distributed energy generation (DEG) has been widely considered as an important ingredient of future smart grid. Energy consumers equipped with the DEG capacities (e.g., by using the distributed solar panels and wind turbines to collect the renewable energy from environments) can also play as the energy suppliers to accommodate their energy demands locally, thus taking advantages in both exploiting renewable energy sources and reducing the energy transmission loss from power plants far way. However, being both a consumer and a supplier, the energy user with DEG capability faces an important question, i.e., how to schedule its DEG output to minimize its energy-provisioning cost. This question becomes even more challenging when a large number of energy users coexist and are allowed to trade energy among them in a cooperative manner. Based on this motivation, we investigate the cooperative DEG and energy trading for future smart grid in this paper.

Optimal energy consumption scheduling to minimize the user's energy-provisioning cost (e.g., via demand-response management) has been an important topic in smart grid [2–5]. In [2], the authors proposed a residential energy consumption scheduling framework which aimed to achieve a desired trade-off between minimizing the electricity expenditure and minimizing the dis-utility of users' electricity appliances. In [3], the authors considered the privacy issue of users' energy scheduling and adopted the mechanism design to achieve a self-incentive based energy consumption scheduling. In [4], the authors adopted a noncooperative game formulation to model different users' energy consumption scheduling and proposed a distributed algorithm to achieve the equilibrium of the game model. The authors of [5] also investigated the distributed energy consumption scheduling problem, and they took account of the influence due to the loss in message exchange. However, the above work did not consider the DEG capability. Nevertheless, there are growing interests in investigating the optimal energy consumption scheduling policy for energy users of DEG capabilities and quantifying the associated benefits from the DEG [6–8]. In [6], the authors proposed a decision-support tool, which incorporated the local DEG sources, to maximize the residential consumer's benefit while guaranteeing its desired energy service. [7] took account of the intermittence of DEG output and investigated the user's corresponding optimal demand scheduling. Our previous work [8] used the technique of stochastic dominance to investigated how the intermittence of DEG output influenced the user's optimal energy consumption scheduling.

Recently, the growing intelligence of power grid and the emerging paradigm of micro-grids provide energy users the new opportunities to trade their temporarily unused energy for benefits [9–13]. In [9], the authors investigated the optimal energy exchange between two micro-grids (with each having the DEG capability) to minimize their total cost resulting from the energy generation and transportation. In [10], via allowing the energy trading among users, the authors proposed a non-collaborative approach and a collaborative approach to maximize individual user's welfare and

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the social welfare, respectively. In [11], the authors investigated the competition among electrical vehicles, which sold part of their stored energy to the power grid. In [12], the authors investigated the coordination of vehicle-to-grid services via energy trading to enhance the grid stability. The authors of [13] used the game models to model the interactions among a number of distributed storage units in their energy trading.

However, to the best of our knowledge, it is still an open question about how each individual energy user can benefit from its cooperative energy scheduling and energy trading with the others. This is our focus in this work. Our key contributions can be summarized as follows: (i) we first propose a joint optimization problem of DEG scheduling and energy trading to minimize the total energy-provisioning cost of all users; (ii) based on the optimal solution of the joint optimization, we further propose a benefit-sharing optimization problem, which determines the transaction costs associated with the users' energy trading, such that each user can positively benefit from its cooperative DEG and energy trading with the others; (iii) extensive numerical results are performed to show the performance of our proposed joint scheduling optimization and the benefit-sharing optimization.

The rest of this paper is organized as follows. We describe the system model in Section 2. To make a benchmark for comparison, we first investigate the model without energy trading in Section 3. Next, in Section 4, we investigate the model allowing the energy trading in depth, with Subsection 4.1 devoted to the joint optimization of DEG and energy trading and Subsection 4.2 devoted to the benefit-sharing optimization. We present extensive numerical results in Section 5 and conclude this work finally in Section 6.

2 System Model

We first briefly introduce the system model in this work. As shown in Figure 1, we consider the system model in which a group of energy users (EUs), denoted by $\mathcal{N} = \{1, 2, ..., N\}$, coexist a residential area. There is an intelligent controller (IC), who controls each EU's energy-acquisition from the macro-grid and the energy trading among the EUs. Figure 2 further illustrates each EU's structure, in which we use the red (and black) solid lines to explicitly denote the *external* (and *internal*) energy flows, respectively. Specifically, each EU is equipped with a smart meter, which receives the decisions on energy scheduling from the IC and performs the corresponding scheduling decisions. As shown in Figure 2, we consider that each EU is equipped with a storage device, which can temporally store its unused energy for future use. Meanwhile, each EU is also equipped with a distributed energy generator (DEG), which can provide energy locally.

2.1 Model of Each EU's Energy Scheduling

We next illustrate the details about each EU's energy scheduling decisions. Specifically, we consider that each EU consists of two subsystems (as shown Figure 2), namely, the demand-provisioning subsystem and the storage subsystem. We describe these two subsystems as follows.

1) *Demand-Provisioning Subsystem:* Each EU's demand-provisioning subsystem serves to meet its local energy demand. Specifically, we consider each EU has a local

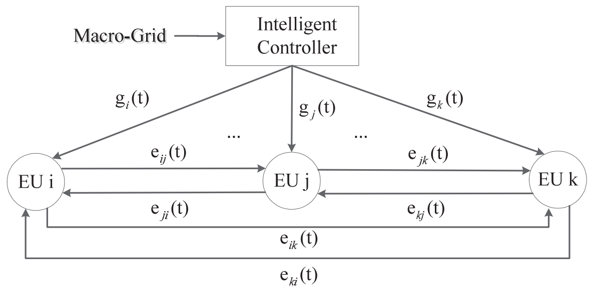

Fig. 1: Illustration of system model

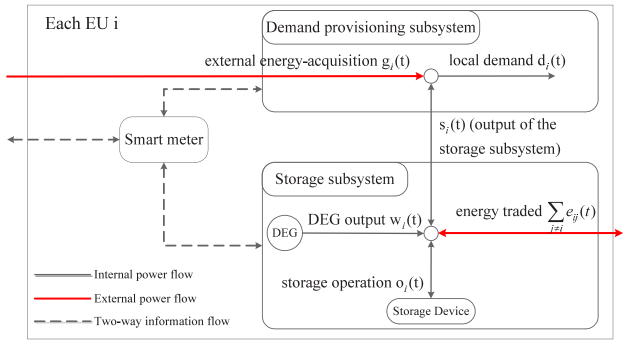

Fig. 2: Illustration of each EU's structure

demand profile, denoted by $\{d_i(t)\}_{t\in\mathcal{T}}$, over the time horizon of interest[1]. To meet its local energy demand, each EU can i) acquire energy from the macro-grid, which is denoted by $\{g_i(t)\}_{t\in\mathcal{T}}$ (with each nonnegative element $g_i(t)$ denoting its energy-acquisition at slot t), and ii) use the output of its storage subsystem, which is denoted by $\{s_i(t)\}_{t\in\mathcal{T}}$. Its objective is to keep the following set of constraints met:

$$s_i(t) + g_i(t) = d_i(t), \forall t \in \mathcal{T}. \tag{1}$$

Notice that $s_i(t) \geq 0$ means that the storage subsystem is providing energy to meet the EU's demand. Otherwise, it means that the storage subsystem is absorbing energy.

2) *Storage Subsystem:* Each EU's storage subsystem serves to regulate its operation on the storage device, its DEG output and its energy trading with the other EUs.

First, recall that each EU has a storage device which can temporally store its unused energy for future use. Let $B_i(t)$ denote the state of EU i's storage at slot t, and let $o_i(t)$ denote the operation (i.e., charging or discharging) on the storage device. Thus, the dynamics of $\{B_i(t)\}_{t\in\mathcal{T}}$ can be expressed as

$$B_i(t+1) = B_i(t) + o_i(t), t \in \mathcal{T}, \tag{2}$$

where $o_i(t) \geq 0$ means that the storage is being charged at slot t, and $o_i(t) < 0$ means the opposite. Due to some physical constraints, the state of the storage device is both lower bounded by its minimum level $B_i^{\min}$ and upper bounded by its maximum level $B_i^{\max}$ at each slot as follows

$$B_i^{\min} \leq B_i(t) \leq B_i^{\max}, \forall t \in \mathcal{T}. \tag{3}$$

[1] In this model, we assume that each EU has an accurate prediction of its energy demand over the time horizon of interest. An interesting future work is to analytically take account of the uncertainty in each EU's demand.

In addition, the operation on the storage is constrained by

$$o_i^{\min} \leq o_i(t) \leq o_i^{\max}, \forall t \in \mathcal{T}, \tag{4}$$

where the negative number $o_i^{\min}$ and the positive number $o_i^{\max}$ denote the maximum discharge rate and the maximum charge rate of the storage device, respectively.

Second, each EU is equipped with a DEG for its local energy generation. Let $w_i(t)$ denote the output of DEG at slot t. It is upper bounded by the maximum generation level $w_i^{\max}$ as follows

$$0 \leq w_i(t) \leq w_i^{\max}, \forall t \in \mathcal{T}. \tag{5}$$

In addition, the total generation of EU's DEG over the entire horizon is upper bounded by a maximum level w_i^{up} as follows

$$\sum_{t \in \mathcal{T}} w_i(t) \leq w_i^{\text{up}}. \tag{6}$$

Third, each EU can trade energy with the other EUs when feasible. We use Ω_i to denote the set of EUs with whom EU i can trade energy. Let $e_{ij}(t)$ denote the energy traded from EU $i \in \Omega_j$ to EU $j \in \Omega_i$ at slot t. Specifically, $e_{ij}(t) \geq 0$ means that EU i sells energy equal to $|e_{ij}(t)|$ to EU j at slot t. Otherwise, it means that EU i buys energy equal to $|e_{ij}(t)|$ from EU j. Please note that $e_{ji}(t)$ holds the similar meaning as $e_{ij}(t)$, and it is from the perspective of EU j. Since the energy trading between EU i and EU j should be mutually compatible, the following set of constraints

$$\begin{aligned} & e_{ij}(t) + e_{ji}(t) = 0, \\ & \{(i,j) \in (\mathcal{N}, \mathcal{N}) | i \in \Omega_j \text{ and } j \in \Omega_i\}, \forall t \in \mathcal{T}. \end{aligned} \tag{7}$$

should be met. Notice that it is reasonable that $j \in \Omega_i$ when $i \in \Omega_j$, i.e., EU i and EU j are mutually accessible.

Considering its operation on the storage device, its DEG output and its trading with the other EUs, each EU's output of its storage subsystem, i.e., $s_i(t)$, can be given by

$$s_i(t) = w_i(t) - o_i(t) - \sum_{j \in \Omega_i} e_{ij}(t), \forall t \in \mathcal{T}. \tag{8}$$

As mentioned below constraint (1), when $s_i(t) \geq 0$, the *net effect* of storage subsystem is providing energy to meet EU i's demand. Otherwise, the net effect of storage subsystem can be considered as absorbing energy with amount of $|s_i(t)|$.

2.2 Model of Each EU's Energy-Provisioning Cost

We quantify each EU's energy-provisioning cost in this subsection. Based on its energy scheduling decisions, each EU's energy-provisioning cost includes three parts, namely, its cost of energy-acquisition from macro-grid, its cost of DEG output, and its cost associated with the energy trading.

1) *Cost of Energy-Acquisition from Macro-grid:* Let $p(t)$ denote the real-time electricity price charged by the macro-grid. Then, each EU i's cost for its energy-acquisition from macro-grid can be expressed as $\sum_{t \in \mathcal{T}} p(t) g_i(t)$.

2) *Cost of DEG Output:* Each EU's DEG output also incurs a certain cost. We use function $G_i^t(w_i(t))$ to denote the associated cost of EU i at slot t when its DEG output is equal to $w_i(t)$. For simplicity, we assume that $G_i^t(.)$ is continuously increasing and strictly convex, meaning that each EU's marginal cost in its DEG output is increasing.

3) *Cost associated with Energy Trading:* Specifically, let us consider the energy trading between two EUs, say EU $i \in \Omega_j$ and EU $j \in \Omega_i$. We use function $C_{ij}^t(.)$ to denote the cost associated with the energy trading between EU i and EU j at slot t. There are two different cases associated with the transaction (i.e., the energy trading) as follows:

Case 1: when $e_{ij}(t) \geq 0$, EU i sells energy to EU j with amount equal to $e_{ij}(t)$ at slot t. In this case, EU i charges EU j the cost equal to $C_{ij}^t(e_{ij}(t))$ at slot t.

Case 2: when $e_{ij}(t) < 0$, EU i buys energy from EU j with amount equal to $|e_{ij}(t)|$. In this case, EU i pays EU j the cost equal to $C_{ij}^t(|e_{ij}(t)|)$ at slot t.

As a summary of the above three parts, each EU's total energy-provisioning cost Q_i can be given by

$$\begin{aligned} Q_i = & \sum_{t \in \mathcal{T}} p(t) g_i(t) + \sum_{t \in \mathcal{T}} G_i^t(w_i(t)) - \\ & \sum_{t \in \mathcal{T}} \sum_{j \in \Omega_i} I(e_{ij}(t)) C_{ij}^t(|e_{ij}(t)|), \end{aligned} \tag{9}$$

where function $I(.)$ is a special function with $I(x) = 1$ if $x \geq 0$, and $I(x) = -1$ otherwise.

3 Analysis of A Benchmark Scenario without Energy Trading among EUs

3.1 Problem Formulation of a Benchmark Scenario

For the sake of comparison, we first consider a benchmark scenario in which the energy trading is not allowed. In this scenario, each EU individually minimizes its energy-provisioning cost Q_i by solving the optimization problem:

$$\begin{aligned} \text{(P1):} \ & \min Q_i \\ \text{subject to:} \ & \text{constraints } (1)(2)(3)(4)(5)(6) \text{ and } (8) \\ & e_{ij}(t) = 0, \{(i,j) \in (\mathcal{N}, \mathcal{N}) | i \in \Omega_j \text{ and } j \in \Omega_i\}, \forall t \in \mathcal{T} \\ \text{variable:} \ & g_i(t), s_i(t), w_i(t), o_i(t), \forall t \in \mathcal{T} \end{aligned}$$

Notice that the above constraint $e_{ij}(t) = 0$ excludes the energy trading among EUs. We use $\{g_i^{\text{non}}(t)\}$, $\{s_i^{\text{non}}(t)\}$, $\{w_i^{\text{non}}(t)\}$ and $\{o_i^{\text{non}}(t)\}$ to denote the optimal solutions of Problem (P1) and use Q_i^{non} to denote the corresponding optimal objective function value, which represents EU i's minimum total cost over the entire horizon without allowing the energy trading. Here, the superscript "non" means the "non-cooperative" case without energy trading.

3.2 Algorithm to Solve Problem (P1)

Facing Problem (P1), we propose an efficient algorithm to solve it. Using constraint (1) and constraint (8), we can substitute $g_i(t)$ and $s_i(t)$ by $w_i(t)$ and $o_i(t)$, and thus equivalently simply Problem (P1) into the following optimization problem (the capital letter "E" means "Equivalence"):

$$
\begin{aligned}
\text{(P1-E):} \ \min \sum_{t\in\mathcal{T}} G_i^t(w_i(t)) - \sum_{t\in\mathcal{T}} p(t)w_i(t) + \\
\sum_{t\in\mathcal{T}} p(t)o_i(t) + \sum_{t\in\mathcal{T}} p(t)d_i(t) \\
\text{subject to: constraints } (2)(3)(4)(5) \text{ and } (6) \\
w_i(t) - o_i(t) \leq d_i(t), \forall t \in \mathcal{T} \\
\text{variables: } w_i(t), o_i(t), \forall t \in \mathcal{T}.
\end{aligned}
$$

The last constraint stems from the constraint that $s_i(t) \leq d_i(t)$, meaning that the output of the storage subsystem cannot exceed the EU's local energy demand. Solving Problem (P1-E) directly might require a high computational complexity, since it involves a joint optimization over vector $\{w_i(t)\}$ and vector $\{o_i(t)\}$.

Fortunately, a close observation of Problem (P1-E) shows that it consists of two sub-problems as follows:

$$
\begin{aligned}
\text{(P1-E-SubA):} \ \min_{\{w_i(t)\}} \sum_{t\in\mathcal{T}} G_i^t(w_i(t)) - \sum_{t\in\mathcal{T}} p(t)w_i(t) \\
\text{subject to: constraints } (5) \text{ and } (6) \\
\text{(P1-E-SubB):} \ \min_{\{o_i(t)\}} \sum_{t\in\mathcal{T}} p(t)o_i(t) \\
\text{subject to: constraints } (2), (3) \text{ and } (4)
\end{aligned}
$$

Subproblems (P1-E-SubA) and (P1-E-SubB) are coupled by the set of constraints $w_i(t) - o_i(t) \leq d_i(t), \forall t \in \mathcal{T}$. Based on these two subproblems, we propose the following Algorithm (A1) to solve Problem (P1-E) (as well as Problem (P1)).

Algorithm (A1) follows the rationale of the *alternative direction method* [17]. Specifically, in each round of iteration, Algorithm (A1) first determines the optimal storage operation by using the fixed DEG output (i.e., Line 3 of Algorithm (A1)). Then, Algorithm (A1) continues to determine the optimal DEG output by fixing the storage operation (i.e., Line 4 of Algorithm (A1)). This process continues until reaching convergence, i.e., the obtained sequences of the optimal storage operation and the DEG output present a negligible change (i.e., Line 5). Using the outcomes of Algorithm (A1) (i.e., $\{w_i^{\text{non}}(t)\}$ and $\{o_i^{\text{non}}(t)\}$ in Line 9) and constraints (1) (8), we can further obtain $\{g_i^{\text{non}}(t)\}$ and $\{s_i^{\text{non}}(t)\}$. Thus, we finish solving Problem (P1). The advantage of Algorithm (A1) lies in that it decomposes the original complicated optimization problem (i.e., Problem (P1-E)) into two subproblems (i.e., Problem (P1-E-SubA) and Problem (P1-E-SubB)) in parallel, both of which can thus be solved with a low computational complexity.

Remark 1: In Line 1 of Algorithm (A1), we initialize $\{\widetilde{w}_i(t)\}$ by first solving Problem (P1-E-SubA) and then randomly disturb it. Our purposes are two-folded, i.e., to start with an initialization which might be close to the optimum and to avoid being trapped by a local optimum due to a fixed initialization. Thanks to the convexity of Problem (P1), the numerical results in Section 5 show that Algorithm (A1) can solve Problem (P1) accurately.

Algorithm (A1) to solve Problem (P1-E)

1: Solve Problem (P1-E-SubA) and denote its optimal solution by $\{\widetilde{w}_i(t)\}_{\forall t\in\mathcal{T}}$. Further disturb $\{\widetilde{w}_i(t)\}$ according to $\widetilde{w}_i(t) = \alpha(t)\widetilde{w}_i(t)$, where $\{\alpha(t)\}$ is a set of random parameters following the identical and independent uniform distribution within [0, 1].
2: **while** 1 **do**
3: Solve Problem (P1-E-SubB) with the additional set of constraints $\widetilde{w}_i(t) - d_i(t) \leq o_i(t), \forall t \in \mathcal{T}$, and denote its optimal solution by $\{\widetilde{o}_i(t)\}_{\forall t\in\mathcal{T}}$.
4: Solve Problem (P1-E-SubA) with the additional set of constraints $w_i(t) \leq \widetilde{o}_i(t) + d_i(t), \forall t \in \mathcal{T}$, and denote its optimal solution by $\{\widetilde{w}_i(t)\}_{\forall t\in\mathcal{T}}$.
5: **if** the obtained $\{\widetilde{o}_i(t)\}$ and $\{\widetilde{w}_i(t)\}$ converge simultaneously **then**
6: Break the WHILE loop.
7: **end if**
8: **end while**
9: Set the optimal solution of Problem (P1-E) as $w_i^{\text{non}}(t) = \widetilde{w}_i(t)$ and $o_i^{\text{non}}(t) = \widetilde{o}_i(t), \forall t \in \mathcal{T}$.

4 Analysis of a Cooperative Model with Energy Trading among EUs

4.1 Joint Optimization of DEG and Energy Trading

We next consider the scenario in which all the EUs are actively involved in energy trading such that their total energy-provisioning cost can be minimized. Specifically, we formulate a joint optimization of the DEG output and energy trading for all EUs as follows:

$$
\begin{aligned}
\text{(P2):} \ \min \sum_{i\in\mathcal{N}} Q_i = \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} p(t)g_i(t) + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} G_i^t(w_i(t)) \\
\text{subject to: constraints } (1)(2)(3)(4)(5)(6)(7) \text{ and } (8) \\
\text{variables: } s_i(t), w_i(t), g_i(t), o_i(t), \forall i \in \mathcal{N} \text{ and } \forall t \in \mathcal{T}, \\
\text{and } e_{ij}(t), \{(i,j) \in (\mathcal{N},\mathcal{N}) | i \in \Omega_j \text{ and } j \in \Omega_i\}, \forall t \in \mathcal{T}.
\end{aligned}
$$

Notice that in the objective function of Problem (P2), the set of energy trading costs $\{C_{ij}^t(|e_{ij}(t)|)\}$ are already canceled out. Let $\{s_i^{\text{co}}(t)\}, \{w_i^{\text{co}}(t)\}$, $\{g_i^{\text{co}}(t)\}$, $\{o_i^{\text{co}}(t)\}$ and $\{e_{ij}^{\text{co}}(t)\}$ denote the optimal solutions of Problem (P2), and further let Q^{co} denote the optimal objective function value, which represents the minimum total cost of all EUs when their energy trading is allowed. The superscript "co" means the "cooperative" case allowing the energy trading. Different from Problem (P1), Problem (P2) couples all EUs' energy scheduling decisions and thus requires a joint optimization over all EUs. Nevertheless, thanks to the convexity of Problem (P2), it still can be successfully solved, e.g., via the interior point method [16]. In particular, we have the following result about the optimum of Problem (P2).

Remark 2: As represented by Problem (P2), the *cooperative model* incorporating energy trading outperforms the *noncooperative model* without energy trading (as represented by Problem (P1)) in terms of achieving a lower total energy-provisioning cost, i.e., $Q^{\text{co}} \leq \sum_{i\in\mathcal{N}} Q_i^{\text{non}}$. The key reason lies in that energy trading yields a greater flexibility for all EUs to schedule their energy-provisioning in an appropriate manner, which consequently achieves a lower *social cost*.

4.2 Formulation of Benefit-Sharing and Its Solution

Recall that the set of energy trading costs are already canceled out in the objective function of Problem (P2). As a result, each individual EU's cost is still to be determined. To this end, it is a reasonable to assume that each EU expects to benefit from the energy trading, i.e., achieving a lower cost in contrast to the case without energy trading. Motivated by this, we formulate a benefit-sharing optimization problem as

$$
\begin{aligned}
\text{(P3): } & \max \prod_{i\in\mathcal{N}} (Q_i^{\text{non}} - \hat{Q}_i) \\
\text{s.t.: } & \hat{Q}_i = \sum_{t\in\mathcal{T}} p(t) g_i^{\text{co}}(t) + \sum_{t\in\mathcal{T}} G_i(w_i^{\text{co}}(t)) - \\
& \quad \sum_{t\in\mathcal{T}} \sum_{j\in\Omega_i} I(e_{ij}^{\text{co}}(t)) C_{ij}^t(|e_{ij}^{\text{co}}(t)|), \forall i \in \mathcal{N}, \\
& Q_i^{\text{non}} \geq \hat{Q}_i, \forall i \in \mathcal{N}, \\
& C_{ij}^t(|e_{ij}(t)|) = C_{ji}^t(|e_{ji}(t)|), \{(i,j) \in (\mathcal{N},\mathcal{N}) | i \in \Omega_j \\
& \qquad \text{and } j \in \Omega_i\}, \forall t \in \mathcal{T}, \\
\text{var.: } & \hat{Q}_i, \forall i \in \mathcal{N}, \text{ and} \\
& C_{ij}^t(.), \{(i,j) \in (\mathcal{N},\mathcal{N}) | i \in \Omega_j \text{ and } j \in \Omega_i\}, \forall t \in \mathcal{T}.
\end{aligned}
$$

In Problem (P3), the set of $\{Q_i^{\text{non}}\}$ are known via solving Problem (P1). Meanwhile, $\{w_i^{\text{co}}(t), g_i^{\text{co}}(t), s_i^{\text{co}}(t), o_i^{\text{co}}(t), e_{ij}^{\text{co}}(t)\}$ are known via solving Problem (P2). The essence of Problem (P3) is to determine the set of cost functions $\{C_{ij}^t(.)\}$ associated with the EUs' energy trading such that: *i) each EU can positively benefit from energy trading, and ii) different EUs' net-benefits from their energy trading are distributed in a fair manner.* Problem (P3) follows the rationale of Nash bargaining game [15], which models how a set of agents of different interests cooperatively share a common yet limited social resource with the objective to make each agent perceive a fair benefit. This objective essentially turns into the so-called *Nash-product* as represented by the objective function of Problem (P3). Here Q_i^{non} denotes EU i's internal threaten point, beyond which EU i will not have any incentive to join the cooperation.

Solving Problem (P3) directly is very difficult, since there are numerous forms of the cost functions, which fit Problem (P3). Nevertheless, we have the following result regarding the optimum of Problem (P3).

Lemma 1. *At the optimum of Problem (P3), the set of cost functions $\{C_{ij}^t(.)\}$ should meet*

$$
\begin{aligned}
& \sum_{j\in\Omega_i} \sum_{t\in\mathcal{T}} I(e_{ij}^{co}(t)) C_{ij}^t(|e_{ij}^{co}(t)|) = \\
& \sum_{t\in\mathcal{T}} p(t) g_i^{co}(t) + \sum_{t\in\mathcal{T}} G_i^t(w_i^{co}(t)) - \\
& \quad \frac{N-1}{N} Q_i^{non} + \frac{1}{N} \sum_{j\in\mathcal{N}, j\neq i} Q_j^{non} - \frac{1}{N} Q^{co}, \forall i \in \mathcal{N}. \quad (10)
\end{aligned}
$$

Proof: Recall that N denotes the total number of EUs. For a sake of clear presentation, let us introduce an auxiliary parameter Λ_i, i.e., $\Lambda_i = \sum_{t\in\mathcal{T}} p(t) g_i^{\text{co}}(t) + \sum_{t\in\mathcal{T}} G_i^t(w_i^{\text{co}}(t))$. Meanwhile, let us treat each EU's energy trading cost as a whole as $m_i = \sum_{j\in\Omega_i} \sum_{t\in\mathcal{T}} I(e_{ij}^{\text{co}}(t)) C_{ij}^t(|e_{ij}^{\text{co}}(t)|)$. Then, the above Problem (P3) becomes equivalent to

$$
\text{(P4): } \max_{\{m_i\}} \sum_{i\in\mathcal{N}} \ln\left(Q_i^{\text{non}} - (\Lambda_i - m_i)\right), \text{ s.t.: } \sum_{i\in\mathcal{N}} m_i \leq 0.
$$

[2] The convexity of the above optimization problem (P4) enables us to use the Karush-Kuhn-Tucker (KKT) conditions to quantify its optimality as follows [16]

$$
\frac{1}{Q_i^{\text{non}} - \Lambda_i + m_i} = \lambda, \forall i \in \mathcal{N}, \quad (11)
$$

where λ denotes the Lagrangian dual price for the constraint $\sum_{i\in\mathcal{N}} m_i \leq 0$, and it is commonly applied to all the EUs. By solving the above set of optimality conditions in (11) and exploiting the property that $\sum_{i\in\mathcal{N}} m_i = 0$ holds at the optimum, we can obtain the results given in (10). Notice that $Q^{\text{co}} = \sum_{i\in\mathcal{N}} \Lambda_i$. This finishes the proof. □

Corollary 1: Based on Problem (P3), all EUs equally share the total net-benefit from energy trading. In other words, via the energy trading, each EU can reduce its total cost by $\frac{1}{N}(\sum_{i\in\mathcal{N}} Q_i^{\text{non}} - Q^{co})$ in comparison with the case without energy trading. This result essentially follows the rationale of the Nash bargaining process, which guarantees a fair allocation of the net-benefit among all agents [15].

Lemma 1 implies that the optimal transaction costs associated with the EUs' energy trading in fact are relatively independent of the detailed choices of function $\{C_{ij}^t(.)\}$. In other words, any set of $\{C_{ij}^t(.)\}$ meeting condition (10) suffices to be the optimal solution of Problem (P3). Despite its simplicity, this result is of practical importance since it yields an easy implementation of the energy trading process. Specifically, each EU will not need to record the detailed selling or buying process with each of its feasible partners. Instead, we can treat the intelligent controller (IC) as the coordinator whom each EU charges with or pays to, namely, each EU only needs to negotiate with the IC about its charge (for providing energy to the other EUs) or its payment (for receiving energy from the other EUs). Based on this rationale, we propose Algorithm (A2), shown on the top of Page 6, to implement this negotiation between the IC and the EUs. In Algorithm (A2), parameter θ is an auxiliary variable, which is equivalent to $\frac{1}{\lambda}$ in (11). A deeper observation of θ shows that it denotes *the consensus of all EUs about their share of the net-gain*. Algorithm (A2) essentially exploits the optimality condition (11), which indicates that each EU's charge (or payment) is monotonically decreasing in λ (or increasing in θ). Based on this property, Algorithm (A2) uses the bisection method to determine an appropriate value of θ, which can yield $\sum_{i\in\mathcal{N}} m_i = 0$. Specifically, in Line 5 and Line 8 of Algorithm (A2), we use $|\sum_{i\in\mathcal{N}} m_i| < \epsilon$ to approximate $\sum_{i\in\mathcal{N}} m_i = 0$ numerically, where ϵ is a very small yet positive number denoting the tolerance for computational error. Therefore, Algorithm (A2) is guaranteed to converge to the optimum of Problem (P4) within $\log_2 \frac{\max_{i\in\mathcal{N}}\{Q_i^{\text{non}} - \Lambda_i\} - \min_{i\in\mathcal{N}}\{Q_i^{\text{non}} - \Lambda_i\}}{\epsilon}$ rounds of iterations.

[2] Please notice that in Problem (P4), we have relaxed the constraint $\sum_{i\in\mathcal{N}} m_i = 0$ by $\sum_{i\in\mathcal{N}} m_i \leq 0$ without causing any loss of the optimality of Problem (P4).

Algorithm (A2) to implement the energy trading process

The IC initializes $\theta^{\max} = \max_{i\in\mathcal{N}}\{Q_i^{\text{non}} - \Lambda_i\}$ and $\theta^{\min} = \min_{i\in\mathcal{N}}\{Q_i^{\text{non}} - \Lambda_i\}$.
while 1 **do**
  The IC sets $\theta = \frac{1}{2}(\theta^{\max} + \theta^{\min})$ and broadcast this θ to all EUs.
  Each EU sets its payment (or charge) $m_i = \theta - (Q_i^{\text{non}} - \Lambda_i)$ and reports its m_i to the IC.
  if The IC finds the received $\sum_{i\in\mathcal{N}} m_i > \epsilon$ **then**
    The IC sets $\theta^{\max} = \theta$ and broadcasts θ to all EUs.
  else
    if The IC finds the received $\sum_{i\in\mathcal{N}} m_i < -\epsilon$ **then**
      The IC sets $\theta^{\min} = \theta$ and broadcasts θ to all EUs.
    else
      Break the WHILE loop.
    end if
  end if
end while
If $m_i \geq 0$, then EU i charges the IC with the amount equal to m_i. Otherwise, EU i pays the IC with the amount equal to $|m_i|$.

5 Numerical Results

In this section, we show the numerical results of the proposed models with and without energy trading. We set the time horizon as $\mathcal{T} = \{1, 2, ..., 24\}$. The electricity price of macro-grid follows as $p(t) = 0.288$ RMB/kWH during the valley-hours $t \in \{1, 2, ..., 8\}$ and $p(t) = 0.568$ RMB/kWH during the peak-hours $t \in \{9, 10, ..., 24\}$. Besides, we use the data from [18] to model two different types of local demand profile (i.e., $\{d_i(t)\}$), one for the residential user and another for the commercial user, as shown in Figure 3. We set the parameters for the storage device as $B_i^{\max} = 6$kWH, $B_i^{\min} = 0.5$kWH, $B_i(1) = 0.5$kWH, $o_i^{\max} = 0.5$kWH and $o_i^{\min} = -0.5$kWH, and we set the parameters for the DEG as $w_i^{\max} = 2$kWH and $w_i^{\text{up}} = 25$kWH. Besides, we consider that each EU's DEG cost function takes the form as $G_i^t(w_i(t)) = \alpha_i^t(w_i(t))^2 + \beta_i^t w_i(t)$, where the sets of coefficients $\{\alpha_i^t\}$ and $\{\beta_i^t\}$ follow the independent and identical uniform distribution within $[0.2, 0.21]$.

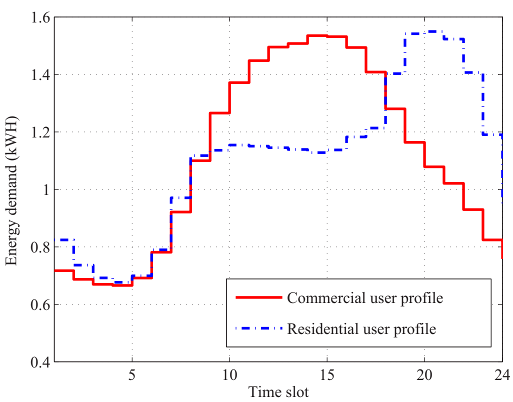

Fig. 3: Two different types of demand profiles.

5.1 Performance of Algorithm (A1) to Solve Problem (P1)

We first show the performance of Algorithm (A1) to solve Problem (P1) in Figure 4 and Figure 5. To set up a benchmark, we first use the CVX (an optimization solver [16]) to solve Problem (P1) and obtain the optimal solution. We then use Algorithm (A1) to solve Problem (P1). In Figure 4, we show the convergence of the optimal solution obtained by Algorithm (A1). The top-plot shows the square-error[3] of the DEG output (i.e., $\{w_i(t)\}$) obtained by Algorithm (A1) against the solution obtained by the CVX. Meanwhile, the bottom-plot shows the square-error of the storage operation (i.e., $\{o_i(t)\}$) obtained by Algorithm (A1) against the solution obtained by the CVX. Both plots show that the square-errors reduce to zero within a very small number of iterations (no more than 15 rounds of iterations, specifically), which means that Algorithm (A1) quickly achieves the optimal solution of Problem (P1). Figure 5 further plots out the convergence of objective function value obtained by Algorithm (A1) (denoted by the solid line marked with circle) in contrast to the value obtained by the CVX solver (denoted by the horizontal dash line). The results again show a fast convergence of Algorithm (A1).

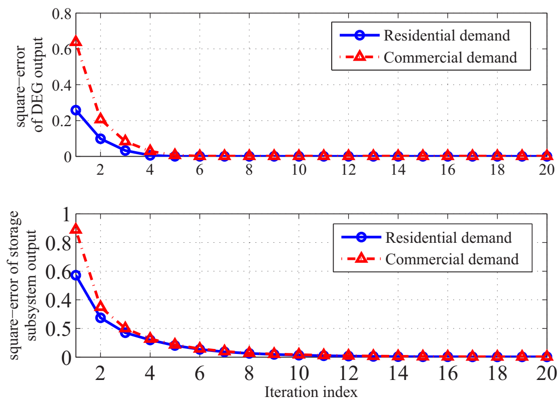

Fig. 4: Convergence of the optimal solution of Alg. (A1).

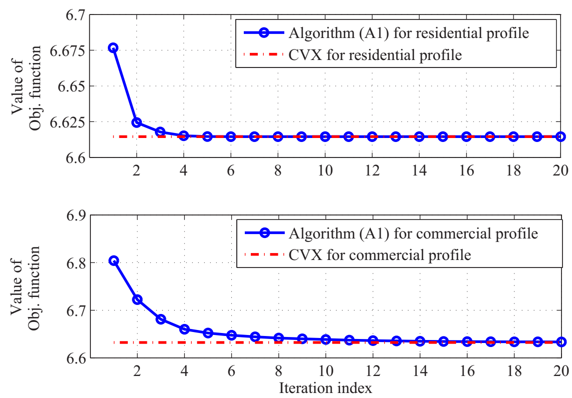

Fig. 5: Convergence of the objective value of Alg. (A1).

Figure 6 illustrates the optimal solutions of Problem (P1)

[3] Specifically, the square-error is given by $\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}(w_i(t) - w_i^*(t))^2$, where $\{w_i^*(t)\}$ denotes the optimal solution from the CVX.

(obtained by Algorithm (A1)) in detail. Specifically, the two plots on the left are for the residential demand profile, and the two plots on the right are for the commercial demand profile. Furthermore, the two plots on the top show the optimal energy scheduling decisions, which include the energy-acquisition $\{g_i(t)\}$ from macro-grid (marked with triangle) and the DEG output $\{w_i(t)\}$ (marked with circle). The two plots in the bottom show the optimal operation of the storage $\{o_i(t)\}$ (marked with star). The results under both demand profiles show that the EU intends to acquire energy from macro-grid to meet its local energy demand and to charge its storage device during the valley-hours. In contrast, during the peak-hours, the EU intends to use its DEG output and to discharge its storage device to meet its local demand (recall that the storage device is charged when $o_i(t)$ is positive, and it is discharged otherwise). These results are consistent with the intuitions well.

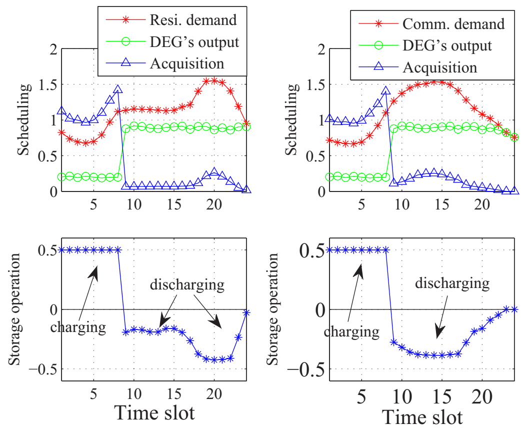

Fig. 6: Optimal scheduling decisions of Problem (P1)

5.2 Performance of the Model with Energy Trading

We next show the performance of the model which allows the EUs to trade energy. In particular, we first consider a scenario of six different EUs (three of them are with the residential demand profile and the rest of them are with the commercial profile). To make the EUs different, we consider that each EU's actual demand is given by $d_i(t)(1+\gamma)$, where γ is a random variable following the uniform distribution within $[-\Gamma, \Gamma]$ and is independent among different EUs (parameter γ can be considered to capture the randomness in the EU's demand). The top-plot in Figure 7 shows each EU's total cost under the energy trading model with one realization of their demands. The results show that each EU can positively benefit from the energy trading in contrast to the model without energy trading. More importantly, as marked out in Figure 7, the net-benefit from energy trading is allocated among different EUs in a fair manner, thus verifying Corollary 1.[4] The bottom-plot in Figure 7 shows each EU's marginal energy price (which is measured by its total cost over its total demand) under the trading model. The result show that the energy trading can help reduce the marginal price experienced by the EU (we also marked out each EU's relatively reduced marginal price with the energy trading in contrast to the model without energy trading).

[4] Although each EU's net-benefit might be small in this illustrative numerical example, the net-benefit of all EUs will be significant under a large population size, which is always true for a practical power grid.

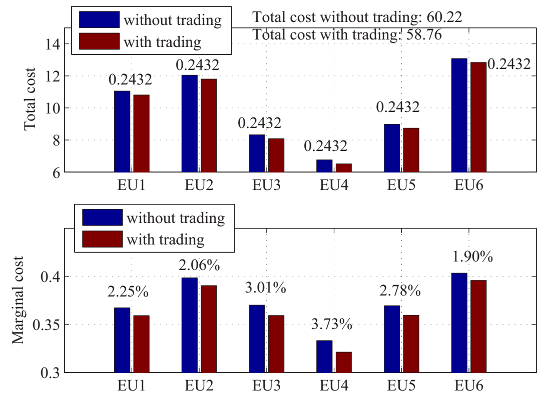

Fig. 7: Performance of the energy trading model under one realization of the EUs' demands (with $\Gamma = 1$)

Table 1 further shows the net-gain of all EUs achieved by the energy trading model under different number of EUs and different values of Γ (i.e., the range of the randomness of each EU's demand). Specifically, each result in Table 1 is averaged over 100 realizations of the EUs' demands. The results show that all EUs can achieve a larger net-gain as the value of Γ increases, which implies that the energy trading model can benefit the EUs more when they have a greater fluctuation in their demands. Moreover, the net-gain from energy trading increases as the number of EUs increases, meaning that the proposed trading model will be very attractive to the practical power grids consisting of a large number of EUs.

Table 1: Average Net-Gain of All EUs under Different Settings

EUs \ Γ	0	0.2	0.4	0.6	0.8	1
6 EUs	0.014	0.022	0.049	0.186	0.484	1.029
9 EUs	0.022	0.037	0.104	0.324	0.832	1.795
12 EUs	0.028	0.049	0.140	0.455	1.134	2.549
18 EUs	0.042	0.076	0.198	0.692	1.791	3.857

Figure 8 plots the optimal scheduling decisions of Problem (P2). To make a clear presentation, we consider two EUs (i.e., with EU1 of the residential demand profile and EU2 of the commercial demand profile) and use the same parameter-setting as in Figure 6. Specifically, the comparison between Figure 8 and Figure 6 indicates that the net-benefit achieved by the energy trading model essentially stems from a greater flexibility in exploiting all EUs' storage devices by allowing their energy trading. In other words, via the trading, all EUs can more wisely schedule the energy-acquisition from the macro-grid and the DEG output to reduce their total energy-provisioning cost efficiently.

Finally, we show the performance of Algorithm (A2) to implement the energy trading in Figure 9. We use the same parameter-setting as in Figure 7. The top-plot of Figure 9 shows the convergence of parameter θ, which in fact represents the consensus of all EUs about their share of the net-gain (as we describe before). The results show that the value

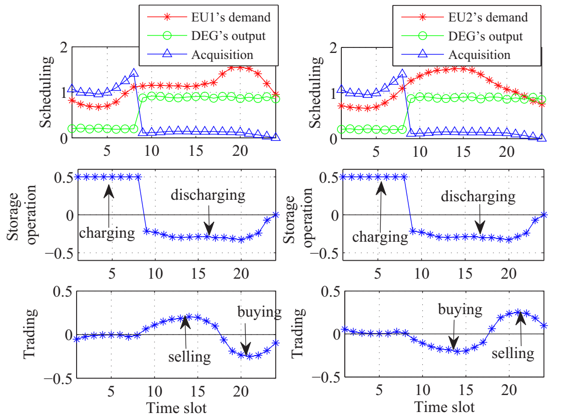

Fig. 8: Optimal scheduling decisions of Problem (P2)

of θ quickly converges to each EU's optimal net-gain (which is obtained as a benchmark by using the CVX to solve Problem (P4)). The bottom-plot of Figure 9 further shows the corresponding convergence process of $\{m_i\}$. In this numerical example, the results show that EU1, EU2 and EU6 pay the IC for their receipt of energy, while the rest of EUs charge the IC for their provisioning of energy.

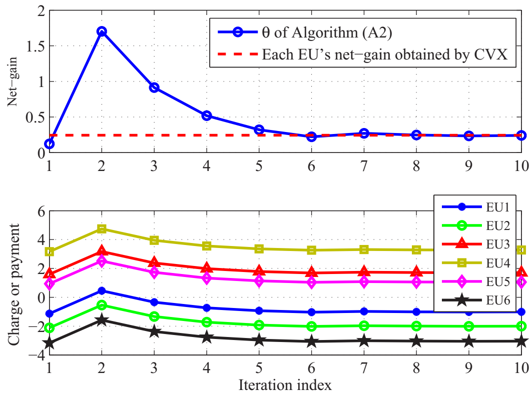

Fig. 9: Algorithm (A2) to implement the energy trading

6 Conclusion

In this paper, we investigate the cooperative distributed energy generation and energy trading for future smart grid. Specifically, we consider a scenario in which a group of energy users, who are equipped with distributed generation capabilities and storage devices, are allowed to trade energy in a cooperative manner with the objective to minimize their total energy-provisioning cost while meeting each individual EU's energy demand. We first jointly determine the optimal energy scheduling decisions for all EUs such that their total energy-provisioning cost can be minimized. Then, based on the optimal energy scheduling decisions, we further determine the optimal charges and payments associated with the EUs' energy trading (i.e., their transactions) to ensure that each individual EU can benefit from the cooperation. Extensive numerical results are provided to validate the advantages of the proposed cooperative distributed energy generation and trading model as well as our proposed algorithms.

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